COORDINATED REPAIR POLICIES FOR A GROUP OF EQUIPMENTS

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CERTIFICATE

Certified that the work on 'Coordinated Repair Policies for a Group of Equipments', by Mr. Pradeep Kumar, has been carried out under my supervision and has not been submitted elsewhere for a degree.

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CONTENTS

ABSTRACT

Coordinated repair scheduling policies are developed for a group of equipments in service producing and goods producing systems. For equipments in a goods producing system, deterioration is manifested by an increase in production cost, whereas for equipments in a service producing system it is indicated by a falling reliability level for that equipment. Maintenance of a goods producing system is undertaken on the principle of both, the minimum operational reliability and minimum cost. For deterministic situations several repair policies-individual, joint and mixed have been presented for both kinds of systems. For all these policies numerical examples are also solved to illustrate the solution procedure. For goods producing systems a stochastic case with chance failures is also considered and a simulation model is developed to solve such repair scheduling problems.

CHAPTER I

INTRODUCTION

1.1 Introduction:

We consider a system comprising of several equipments, each equipment, in turn may comprise of several components. The issue of repair and maintenance of the equipments and their components is vital from the viewpoint of an efficient utilization of the system. All equipments deteriorate in their performance with age. Repair and maintenance are performed to restore the operating characteristics of an equipment to its original or some other specified level. The deterioration of equipments may be broadly classified into two categories. In the first category, the nature of deterioration is known with certainity, whereas, in the second category deterioration is due to chance causes and the nature of deterioration is described in probabilistic terms. In the probabilistic deterioration, the level of deterioration is a random variable, indicating the possibility of failure at any point of time when the level of deterioration exceeds some particular limit and the equipment either becomes completely unfunctional or remains functional but at an undesirable economic condition. Owing to above mentioned characteristic of

deterioration the first category is commonly known as working system while the second category is known as standby system. In practice the deterioration may be a combination of the two types mentioned above. In this study we shall consider, working system, where the nature of deterioration is deterministic.

A system of equipments may be either service producing or goods producing. A convenient measure of deterioration in goods producing system is unit production cost and in service producing system is reliability level.

A common problem related to repair and maintenance of a system of equipments is to find the times and types of repairs (if more than one), to be performed for each member equipment of the system, such that the overall operating cost of the system is a minimum. The solution to this problem, most often, is obtained in presence of some constraints indicating the technological facts and/or managerial policies.

1.2 Certain Concepts:

In order to further describe the system we introduce certain repair and maintenance related concepts, used in the present study.

1.2.1 Major and Minor Repairs:

Whenever an equipment deteriorates, that is the unit

production cost for the equipment in a goods producing system goes up or the reliability level of the equipment in service producing system goes down, a repair operation is called for. The repair operations can be viewed to be of two types. A major repair brings the equipment to 'as good as new' condition whereas a minor repair improves the condition of the equipment but does not restore the equipment to its 'as good as new' condition. The degree of improvement in the system, brought by a minor repair could be understood in terms of the effective age of the equipment, before and after the repair. This will be elaborated while describing the concept of effecting age in sub-section 1.2.3. Minor repairs for all the equipments take place independently, whereas the major repairs for one or more equipments can take place, simultaneously under a common set-up. By performing the maintenance on several equipments simultaneously, we could save on total fixed cost associated with separate repairs of each equipment.

1.2.2 Individual Mixed and Joint Repair Policies:

In many situations, there exists a common component of major repair cost, which can be shared by all those equipments, which have their major repairs simultaneously. When this common cost component is zero, we call the repair policy as individual repair policy. In individual repair policy all the equipments can be treated independently for

obtaining the optimal repair schedule for the system. In case when above mentioned common component is nonzero then the repair policy is termed as mixed repair policy. For 'mixed repair policy' the coordinated set-ups, for repairs, are made at an interval of time, which is equal to the time between two consecutive set-ups of one of the equipments. A 'joint repair policy' is special case of mixed repair policy when the time interval between two consecutive major repairs is same for all the equipments, that is, all the equipments are forced to have their major repairs simultaneously.

1.2.3 Effective Age:

Effective age of an equipment is zero right after a major repair. This would indicate that after a major repair, any equipment in production system will be as good as new as far as the production cost is concerned and any equipment in a service producing system will be as good as new as far as the reliability level of the equipment is concerned. Effective age of an equipment at any instant between two consecutive major repairs, is the effective age at the end of last repair, major or minor, plus the time elapsed after the end of that repair. Thus the effective age just after a minor repair is equal to the effective age just after the previous repair (minor or major) plus u/β, where u is the total time duration during which the equipment has operated between the two repairs, and β is the improvement

factor, defined in next subsection for that repair. From the Fig. 1.1, the effective age of an equipment at any instant 't' after the last major repair is given as:

$$t_e = \frac{X_1 - d_m}{\beta_1} + \frac{X_2 - X_1 - d_1}{\beta_2} + \frac{X_3 - X_2 - d_2}{\beta_3} + \dots + \frac{X_n - X_{n-1} - d_{n-1}}{\beta_n} + (t - X_n - d_n)$$

where X_i is the time after the most recent major repair till the beginning of i-th minor repair, d_i is the duration of i-th minor repair, and n is the number of minor repairs which have taken place before the instant 't'.

1.2.4 Improvement Factor:

Any repair either minor or major brings some improvement in the system. The improvement in the condition of a equipment can be understood either as decrease in the production cost per good unit for the equipment in goods producing system or as an increase in survival probability of a equipment in service producing system. One way of interpreting the above improvement is to think that the 't' year old equipment is no longer that old and its effective age after the repair has reduced to t/β only, where β is the improvement factor. The range for β is from 1 to ∞. $\beta = 1$ corresponds to no improvement in the equipment and $\beta = \infty$ corresponds to the situation where the equipment behaves as a perfectly new equipment. It has been assumed that the above

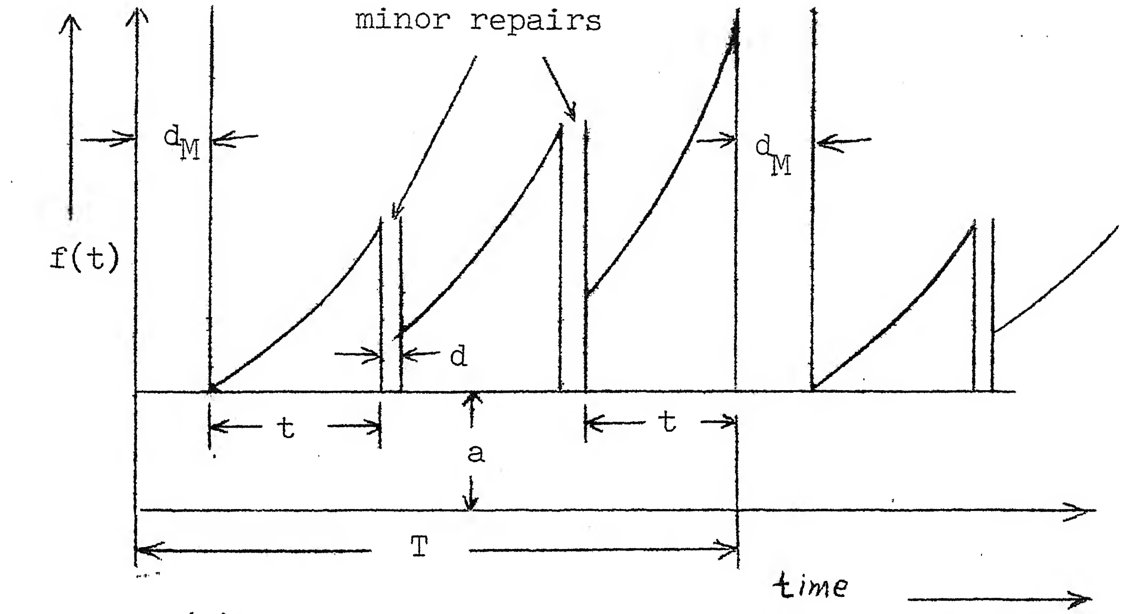

Fig. 1.1(a): An equipment in a goods producing system.

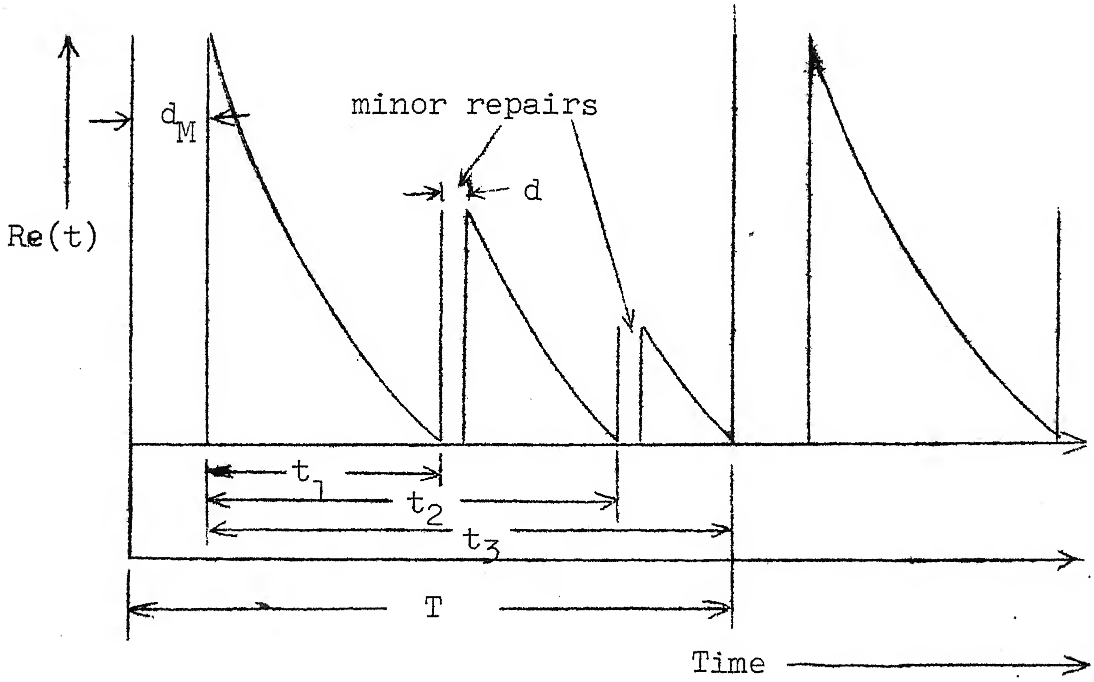

Fig. 1.1(b): An equipment in a service producing system.

reduction in equipment's life takes place only for that time interval for which the equipment has operated after the most recent repair.

For a major repair the improvement factor is infinity, that is, the effective age of the equipment after a major repair is zero. For our present work, it is assumed, that the improvement factor for all the minor repairs of a equipment, is constant with time.

1.2.5 Basic Time Period:

The time period for any equipment is the duration of time interval between two consecutive major repairs of that equipment. For a mixed repair policy the basic time period for the system is equal to the time period of that equipment which has most frequent major repairs. For a joint repair policy, the time periods for all the equipments are equal to the basic time period for the system. For an individual repair policy all the equipments will be having their different time periods. These will, in general, be independent of each other. The concept of basic time period, therefore, has got no meaning under such circumstances.

It is to be noted that the basic time period is not the time interval, after which the whole system repeats its cycle, but it is the time interval after which, the equipment with most frequent major repairs repeats its cycle. The

cycle time for the whole system is an integer multiple of this basic time period.

1.2.6 Basic Operation Duration:

In the present work, we will be using the terms, basic operation duration and the operation duration for an equipment, interchangeably. Operation duration is the time period for which an equipment remains in operating state between two consecutive repairs of that equipment. We will be assuming (in Chapters II and III) that the operation duration for an equipment in goods producing system always remains constant. For service producing system, the operation duration for an equipment keeps decreasing from the first minor repair till the last minor repair.

1.3 Literature Survey:

The problem of repair and maintenance has attracted the attention of many researchers and practioners. There is a vast difference between the practitioners and the researchers in this area. The practitioners basically use their common sense for planning their repair schedules, whereas the researchers usually develop mathematical models for repair schedules, under certain assumptions, which, many a time, are far from reality. Several practices developed and used by practitioners might not have found place in professional journals. A review of literature reveals that many mathematical models have been developed to find the optimal

solution to the problem. A number of researchers, Luss and Hanan [3], Davidson, D. [6] and many others have presented models mainly to solve the problem of a single equipment case where the repair durations have been assumed negligible compared to the operation durations. Luss and Kander [8] and Sule and Harman [1], have developed for a group of equipments, a model for obtaining coordinated maintenance schedule. The model developed by Luss and Kander examines policy for developing coordinated maintenance schedule for identical equipments, having exponential failure rates. On the other hand, the paper by Sule and Harman does not require all the equipments to be identical and thus it allows the operation cost and repair costs for all the equipments to be different from each other.

For service producing systems various preventive maintenance policies have been discussed in detail by Flehinger [7]. For maintenance of service producing systems, stress is mainly laid upon the operational reliability of each individual equipment of the system and of the system as a whole. This is because when a system produces service, the revenue of the service produced can not be realistically calculated. Difficulty with these systems becomes even more severe when such failures endanger human life. Under such circumstances it is desirable, not to allow the reliability of each equipment (or the system) fall below an

acceptable level. Besides Flehinger, there have been some more models developed for maintenance of standby systems [Malik [4, 5,] and for working systems [Malik [2]], but they deal only with the reliability aspects of the system. These models deal only with one-equipment system and that also under the assumption that repair durations are negligible compared to operation durations.

In our present work, we have presented a model for developing repair schedule for a system of non-identical equipments. The repair durations for these equipments are not assumed to be negligible compared to the operation durations between the repairs. For service producing systems, we have developed models for obtaining repair schedule under given reliability constraints. It has been assumed that the reliability constraints of the system can be translated in terms of reliability constraints of individual equipments independently.

1.4 Organization of Thesis:

In Chapter II and III, the mathematical formulations and solution procedures for obtaining the repair schedule for goods producing systems are presented. The treatment has been offered for deterministic systems. The objective is to minimize the total cost per unit time for the system. The total cost is the sum of production cost, repair cost and down-time cost . The production cost during a period is

obtained by integrating the production cost function over that interval. The major and minor repair costs depend upon the number of such repairs carried out during a given interval. The down time cost is realized due to the non-operating mode of the equipment, while a repair is going on. This cost is computed on the basis of duration of down time, irrespective of the kind of repair that is going on at that time. We will neglect the discounting considerations. In Chapter II those systems of equipments have been analysed, for which all the equipments are repaired individually and independently. In Chapter III mixed repair policies have been discussed, where a part of repair cost (fixed cost) can be shared by all the member equipments of the system.

Mathematical formulation and solution procedures for service producing systems have been presented in Chapter IV. All individual, mixed and joint repair policies for deterministic situations have been considered. The objective is to minimize the total cost under given reliability constraints for the system.

Chapter V deals with the repair scheduling of those goods producing systems for which the repair durations for both, minor and major repair, are random. The repair durations are assumed to have known distributions with known parameters. Since chance failures are also considered in

this case, the repair schedule is not concerned, merely with the repairs for preventive maintenance but also with unplanned (emergency) maintenance. Since any mathematical formulation for such system is extremely difficult, we resort to simulation techniques for obtaining the optimal repair schedule.

At the end of Chapter III and IV scope for further studies has been outlined for goods producing and service producing systems, respectively. At the end of Chapter V, we have outlined the scope for future studies for stochastic case of goods producing systems. Chapter VI consists of parametric analysis for individual repair policy in a goods producing system.

Some conclusions have been presented in Chapter VII.

CHAPTER II

INDIVIDUAL REPAIR POLICY

2.1 Introduction:

We consider a machine shop where several reparable equipments are operating independently. Our basic objective is to develop mathematical models for repair policies for this system. In this chapter we present the analysis for the situation, when each equipment is treated independently. This analysis would be directly applicable to the situations where there is no common major set-up cost which can be shared by two or more equipments when repaired simultaneously. Also the result of this study would perhaps be useful to study the joint and mixed repair policies in terms of determining bounds on the decision variables or on the objective function.

For the situation described above we want to minimize the total cost per unit time. The total cost is the sum of production cost, repair cost and down time cost. The decision variables include the number of major repairs per unit time and the number of minor repairs between two consecutive major repairs. As we are considering the deterministic situation, the number and spacing of minor repairs between two consecutive major repairs will remain the same. It is obvious

that as the number of repairs per unit time goes up, the down time cost and the repair cost will go up but the production cost will come down.

We first consider the general case of nonlinear production cost function and nonzero repair time in section (2.3). Two solution procedures are suggested for obtaining the optimal policy of repairs for this general case. In section (2.4) special case with zero repair durations and linear production cost function as discussed. Solution procedures have also been suggested for this case. A representative numerical example has also been solved to illustrate the methodology of the general problem discussed in Sec. (2.3).

2.2 Assumptions and Notations:

2.2.1 Assumptions:

In order to further characterize the system we make following assumption:

1) As the equipments are operating independently the production cost function for one equipment is independent of that of others.

2) Production cost per unit is deterministic and is function of only the effective age of the equipment.

3) There are ample repair facilities so that none of the equipments have to wait for getting repaired.

4) Repair times for minor and major repairs are deterministic and they remain constant with time.

5) Repair costs for minor and major repairs are also deterministic and they also remain constant with time.

6) Downtime cost is deterministic and depends only on the duration of downtime, irrespective of the kind of repair. Downtime cost varies linearly with duration of downtime.

7) Any major repair brings the equipment to 'as good as new' condition. Any minor repair reduces the effective age of the equipment by a constant factor (improvement factor) i.e. the improvement factor for any minor repair is same.

$$\beta_1 = \beta_2 = \beta_3 = \beta_4 = \ldots = \beta_N = \beta \text{ (say)}$$

where β_i is the improvement factor for i-th minor repair.

8) Between two consecutive major repairs all the minor repairs are spaced in such a way that the operation duration between any two repairs is constant, that is, the total operation duration between two consecutive major repairs is divided in $(N + 1)$ equal intervals. N is the number of minor repairs between two consecutive major repairs.

2.2.2 Nomenclature:

The following notations are used:

M Number of the equipments in the system

For j-th equipment:

P_j Minor repair cost

AS_j Cost for one major repair

R_j Downtime cost rate (per unit time)

d_j Duration of minor repair

d_M Duration of major repair

T_j Time between two consecutive major repairs. This will also be termed as Time period .

t_j Basic-operation-duration. This is operation duration between any two consecutive repairs.

N_j Number of minor repairs between two consecutive major repairs.

β_j Improvement factor for minor repair.

$f_j(t)$ Production cost per good unit when t is the effective age of the equipment

$$= a_j + b_j \, t^{n_j}$$

Here a_j, b_j and n_j are constants and represent the characteristics of the equipment.

2.3 Problem Formulation and Solution Procedure:

2.3.1 Problem Formulation:

The sequence of events for an equipment j is shown in the Fig. 1.1(a) in Chapter I. From the figure, the relationship among N_j, t_j, T_j, d_j and d_M can be derived easily.

$$(N_j + 1)\, t_j + N_j\, d_j + d_M = T_j$$

$$t_j = \frac{T_j - N_j\, d_j - d_M}{(N_j + 1)}$$

Let DN_j be given as:

$$DN_j = d_M + N_j . d_j$$

= Downtime duration between two consecutive major repairs for equipment j.

Now the expression for t_j becomes,

$$t_j = \frac{T_j - DN_j}{N_j + 1} \tag{2.1}$$

Since the operation duration between any two consecutive major repairs for equipment j, is assumed to be constant, the effective age of the equipment at any time x after last major repair is given as:

$$t = \begin{cases} x & \text{for } x \leq t_j \\ \dfrac{t_j}{\beta_j} + (x - t_j - d_j) & \text{for } t_j + d_j \leq x \leq 2t_j + d_j \\ \dfrac{2t_j}{\beta_j} + (x - 2(t_j + d_j)) & \text{for } 2(t_j + d_j) \leq x \leq 3t_j + 2d_j \\ \cdot & \\ \cdot & \\ \cdot & \\ \dfrac{N_j t_j}{\beta_j} + [x - N_j(t_j - d_j)] & \text{for } N_j[t_j + d_j] \leq x \leq (N_j + 1)t_j + N \end{cases}$$

Effective age at the beginning of K-th minor repair after a major repair is given by:

$$t_{K,O} = \frac{(K-1)\ t_j}{\beta_j} + t_j$$

Effective age at the end of the K-th minor repair after a major repair is given by:

$$t_{K,1} = \frac{Kt_j}{\beta_j}$$

The production cost between K-th and (K+1)-th minor repair is given by,

$$PC_{K,K+1} = \int_{t_{K,1}}^{t_{K+1,0}} (a_j + b_j\, t^{n_j})\, dt$$

$$= \int_{\frac{Kt_j}{\beta_j}}^{\frac{Kt_j}{\beta_j} + t_j} (a_j + b_j\, t^{n_j})\, dt$$

$$= a_j\, t_j + b_j\, \frac{t_j^{n_j+1}}{n_j+1} \left[\left(\frac{K}{\beta_j} + 1 \right)^{n_j+1} - \left(\frac{K}{\beta_j} \right)^{n_j+1} \right]$$

Production cost per unit time for equipment j can now bex written as:

$$PC_j = \frac{1}{T_j} \sum_{K=0}^{N_j} PC_{K,K+1}$$

$$= \frac{1}{T_j} \left[a_j\, t_j (N_j+1) + \frac{b_j\, t_j^{n_j+1}}{n_j+1} \sum_{K=0}^{N_j} \left[\left(\frac{K}{\beta_j} + 1 \right)^{n_j+1} - \left(\frac{K}{\beta_j} \right)^{n_j+1} \right] \right] \qquad (2.2)$$

Substituting for t_j from Eqn. (2.1), we get,

$$PC_j = \frac{1}{T_j} [a_j (T_j - DN_j) + \frac{b_j}{n_j+1} (\frac{T_j - DN_j}{N_j+1})^{n_j+1}$$

$$\sum_{K=0}^{N_j} \{(\frac{K}{\beta_j} + 1)^{n_j+1} - (\frac{K}{\beta_j})^{n_j+1} \}] \qquad (2.2)$$

Similarly other components of the total cost can be obtained as follows:

$$\text{Repair cost} = A_j + N_j P_j \quad \text{per cycle}$$

$$\text{Repair cost} = \frac{A_j + N_j P_j}{T_j} \quad \text{per unit time}$$

$$\text{Downtime cost per unit time} = \frac{1}{T_j} (d_M + d_j N_j) R_j$$

$$= \frac{DN_j . R_j}{T_j}$$

For equipment j the total cost per unit time is given as:

$$TC_j = PC_j + RC_j + DC_j$$

$$= \{ \frac{A_j + N_j P_j + DN_j . R_j}{T_j} + a_j \frac{T_j - DN_j}{T_j}$$

$$+ \frac{b_j}{n_j+1} \frac{(T - DN_j)^{n_j+1}}{(N_j+1)^{n_j+1}} \cdot \frac{1}{T_j} \sum_{K=0}^{N_j} [(\frac{K}{\beta_j} + 1)^{n_j+1}$$

$$- (\frac{K}{\beta_j})^{n_j+1}]\}$$

$$= \frac{A_j + N_j P_j + DN_j (R_j - a_j)}{T_j} + a_j$$

$$+ \frac{b_j}{n_j+1} \frac{(T-DN_j)^{n_j+1}}{(N_j+1)^{n_j+1}} \frac{1}{T_j} \sum_{K=0}^{N_j} [(\frac{K}{\beta_j} + 1)^{n_j+1} - (\frac{K}{\beta_j})^{n_j+1}]$$

Let us define,

$$C_{2j} = A_j + N_j.P_j + DN_j \ (R_j-a_j)$$

$$C_{1j} = \frac{b_j}{n_j+1} \cdot \frac{1}{(N_j+1)^{n_j+1}} \sum_{K=0}^{N_j} [(\frac{K}{\beta_j} + 1)^{n_j+1} -(\frac{K}{\beta_j})^{n_j+1}]$$

Substituting these in the expression for total cost, we obtain:

$$TC_j = \frac{C_{2j}}{T_j} + a_j + \frac{(T_j - DN_j)^{n_j+1}}{T_j} C_{1j} \qquad (2.3)$$

Careful examination of the above equation leads to the **following inferences.**

1) Out of the two decision variables (N_j and T_j), N_j can have only integer values whereas T_j can have any value on the continuous scale between DN_j and infinity.

2) DN_j, C_{1j} and C_{2j} are functions of N_j only.

3) For a given value of N_j, the total cost expression TC_j is convex in T_j, but nothing can be said about the behaviour of this function with respect to T_j and $N_j^*(T_j)$, where $N_j^*(T_j)$ is the optimal value of N_j for given T_j.

2.3.2 Solution Procedure:

Following two solution procedures are suggested to obtain the optimal values of decision variables N_j and T_j.

1. Procedure I (Exhaustive Search):

When it is expected that the number of minor repairs between two consecutive major repairs is not very large then a thorough search can be made for all possible values of N_j. For every value of N_j (say, from 0 to 5 or 7, this upper limit is usually decided intuitively). We find out the optimal $T_j^*(N_j)$ and the corresponding total cost $TC_j(N_j, T^*(N_j))$. Now we vary N_j for all the integer values, in the expected range, and obtain the corresponding $T_j^*(N_j)$ and $TC[N_j, T_j^*(N_j)]$. We then select the best value of N_j, the corresponding $T_j^*(N_j)$ and $TC(N_j, T_j^*(N_j))$ as the optimal solution for the problem. For given N_j, the optimal value of T_j can be obtained by solving the resulting equation by setting the derivative of TC_j with respect to T_j equal to zero.

$$\frac{\partial TC_j}{\partial T_j} = 0$$

$$\Rightarrow \quad - C_{2j} + C_{1j}\,(T_j^* - DN_j)^{n_j}\,(n_j T_j^* + DN_j) = 0$$

This equation will always give the value of T_j corresponding to the minimum value of the objective function because $\partial^2 TC_j/\partial T_j^2$ can be seen to be positive throughout

the feasible range of T_j. Above equation can not be solved explicitely to yield T_j^* in terms of DN_j, t_j, C_{2j}, C_{1j}; however, some iterative techniques can be used to obtain T_j^*.

The total cost expression can now be written as:

$$TC_j(N_j, T^*(N_j)) = \frac{C_{1j}}{T_j^*} (T_j^* - DN_j)^{n_j} (n_j + 1) T_j^* + a_j$$

$$= C_{1j} (T_j^* - DN_j)^{n_j} (n_j + 1) a_j$$

The solution procedure can now be stated in following steps:

1. Select a particular value of N_j (usually the least in the feasible range or 0).
2. Obtain the optimal $T_j^*(N_j)$ using the iterative method. Calculate corresponding total cost for equipment j.
3. Select next value of N_j (usually one higher than the previous one). If it exceeds the intuitive limit of N_j then go to 4. Otherwise go to 2.
4. Finally, select that value of N_j and corresponding $T_j^*(N_j)$ which gives minimum value of total cost for equipment j.

2. Procedure II:

This solution procedure differs from the exhaustive search procedure in following ways:

1. In the exhaustive search method, we make the primary search along the integer variable N_j, whereas in this

the feasible range of T_j. Above equation can not be solved explicitely to yield T_j^* in terms of DN_j, t_j, C_{2j}, C_{1j}; however, some iterative techniques can be used to obtain T_j^*.

The total cost expression can now be written as:

$$TC_j(N_j, T^*(N_j)) = \frac{C_{1j}}{T_j^*} (T_j^* - DN_j)^{n_j} (n_j + 1)\ T_j^* + a_j$$

$$= C_{1j}\ (T_j^* - DN_j)^{n_j}\ (n_j + 1)\ a_j$$

The solution procedure can now be stated in following steps:

1. Select a particular value of N_j (usually the least in the feasible range or 0).
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4. Finally, select that value of N_j and corresponding $T_j^*(N_j)$ which gives minimum value of total cost for equipment j.

2. Procedure II:

This solution procedure differs from the exhaustive search procedure in following ways:

1. In the exhaustive search method, we make the primary search along the integer variable N_j, whereas in this

procedure we make the primary search along the continuous variable T_j.

2. In the exhaustive search method, since the search is made along the integer variable N_j, we can not miss the optimal solution. However, flexibility of the solution procedure is seriously limited because we cannot make some saving in computational efforts by compromising with non-optimality (slight) of the final solution. On the other hand for the second solution procedure the computational efforts can be reduced considerably by incurring the danger of slight non-optimality in the final solution.

In the present solution procedure, we divide the entire range of T_j in many intervals such that in each interval our objective function TC_j $(T_j, N_j^* (T_j))$ is convex in T_j. This tantamounts to dividing the entire range of T_j in sub-intervals such that $N_j^* (T_j)$ remains same for all the values of T_j in that interval. Now for these sub-intervals, we can find the local optimal values of the decision variables using analytical techniques. Now the global optimal solution for the objective function can be obtained just by selecting the best of all such local optimal solutions.

This procedure can be stated in the following steps:

<u>Step 1</u>: (Dividing the feasible range of T_j)

Start with the lowest feasible value of T_j. Keep varying the value of T_j and finding the value of

$N_j^*(T_j)$ for each value of T_j. That range of T_j in which $N_j^*(T_j)$ does not vary can be considered as one subrange. Thus the entire feasible range of T_j will get divided in several subranges.

Step 2: For each subrange find out the optimal value of T_j^* and calculate the value of total cost for this T_j^* and $N_j^*(T_j)$.

Step 3: Select the best N_j and the corresponding T_j^* which gives the least value of the objective function.

Note: In most of the cases Procedure II will turn out to be noneconomical as compared to the exhaustive search procedure but it has been mentioned here for the sake of completeness.

Computational efforts for the exhaustive search procedure can be reduced considerably, if we have some idea about the probable range of the optimal value of N_j^*. This is because, as a result of this prior information, our search will be restricted to this probable range of optimal values of N_j only. On the basis of several examples solved using Procedure I, it has been found that some initial approximation for N_j^* can be obtained as following:

Let,
$$y_{1j} = AS_j + d_M . R_j$$
$$Z_{1j} = P_j + d_j . R_j$$

If
$$\beta_j \geq \frac{y_{1j}}{Z_{1j}} (\beta_j - 1) \quad \text{then } N_j^* = 0$$

$$\text{If} \quad \beta_j < \frac{y_{1j}}{Z_{1j}}(\beta_j-1) \quad \text{then} \quad (N_j^* + 1) = [(\beta_j-1)(\frac{y_{1j}}{Z_{1j}} - 1)]^{1/}$$

Above value of N_j^* can be used as initial approximation for the optimal value of N_j. It can be seen that y_{1j} represent total cost for one major repair and Z_{1j} represents total cost for one minor repair.

2.4 Numerical Example:

To illustrate the solution procedure, we take up a numerical example. Let us consider a system which has following characteristics.

Number of equipments = 3

Table 2.1: Input data for numerical example.

Parameters	Equipment 1	Equipment 2	Equipment 3
1. Duration of major repair	0.6	0.6	0.6
2. Cost of one minor repair	30.0	20.0	10.0
3. Cost of one major repair	50.0	70.0	90.0
4. Downtime cost rate	60.0	100.0	140.0
5. Duration of one minor repair	0.30	0.20	0.10
6. Improvement factor	5	4	3
7. Unit production cost function	$5+2t^2$	$5+4t^2$	$5+6t^2$

t = effective age of the equipment.

As we have already stated that Procedure I is almost always much more economical than Procedure II, we will follow Procedure I to obtain the optimal solution.

We have defined basic operation-duration as the operation duration between two consecutive repairs. Now onwards, we will be referring this basic operation duration for equipment j as simply operation duration for equipment j . The nature of cost function for one of the equipments (equipment 1) has been shown in Fig. (6.2). Under the assumptions mentioned in Sec. (2.2.1) we can treat each of the equipments individually for obtaining the optimal solution for the system as a whole.

Let us first consider equipment 1.

Let, $N_1 = 0$, the optimal T_1^* is given by following equation:

$$-C_{21} + C_{11}\,(T_1^* - DN_1)^{n_1}\,(n_1 . T_1^* + DN_1) = 0$$

where, $DN_1 = d_M = 0.06$

$$C_{21} = AS_1 + d_M . R_1 = 50 + 0.6 \times 60 = 86$$

$$C_{11} = \frac{b_1}{n_1 + 1} = \frac{2}{2+1} = 0.67$$

Above equation can be solved iteratively to give the optimal value of T_1^* for $N_1 = 0$ as:

$$T_1^* = 4.314$$

The objective function value corresponding to this solution is obtained from the following equation:

$$TC_1^*(N_1, T(N_1)) = C_{11}(T_1^* - DN_1)^{n_1}(n_1+1) + a_1$$

$$TC_1^*(N_1) = 32.16$$

Similarly, we can calculate the optimal $T_j^*(N_j)$ and $TC_j^*(N_j, T_j^*(N_j))$ for all values of N_j and for all the three equipments. These values are given in Table (2.2).

Table 2.2: Optimal T_j^* and TC_j^* for each N_j.

Number of minor repairs (N_j)	Equipment 1		Equipment 2		Equipment 3	
	T_1^*	TC_1^*	T_2^*	TC_2^*	T_3^*	TC_3^*
0	4.314	32.16	3.981	49.85	3.855	67.25
1	7.104	30.85	5.999	44.14	5.293	57.00
2	9.398	31.54	7.528	43.52	6.230	54.59
3	11.349	32.74	8.774	44.29	6.931	54.35
4	13.049	34.11	9.834	45.57	7.500	54.99
5	14.562	35.53	10.765	47.08	7.986	56.02
6	15.929	36.95	11.601	48.58	8.417	57.26

Now we can select the optimal value of N_j (for all the values of j) and the corresponding $T_j^*(N_j)$ and $TC_j^*(N_j, T_j^*(N_j$
The optimal solution is given below:

$$N_1^* = 1 \qquad T_1^* = 7.104 \qquad t_1^* = 3.102$$

$$N_2^* = 2 \qquad T_2^* = 7.528 \qquad t_2^* = 2.176$$

$$N_3^* = 3 \qquad T_3^* = 6.931 \qquad t_3^* = 1.5078$$

Total cost for optimal values of the decision variables.

$$TC_1^*(N_1^*, T_1^*) = 30.85$$
$$TC_2^*(N_2^*, T_2^*) = 43.52$$
$$TC_3^*(N_3^*, T_3^*) = 54.35$$

Total cost for the system can be obtained by adding the three costs.

Total cost for the system = 128.72.

2.5 A Special Case: Negligible Repair Durations and Linear Production Cost Function

Besides the assumptions of Sec. (2.2) we further assume that the repair durations are negligible compared to the operation durations and the unit production cost for any equipment varies linearly with effective age of that equipment.

$$d_M = 0$$
$$d_j = 0 \qquad \text{for all } j$$
$$n_j = 1 \qquad \text{for all } j$$

Unit production cost function can be written as.

$$f_j(t) = a_j + b_j\, t$$

The variables C_{1j}, C_{2j} and DN_j are given as:

$$DN_j = 0$$

$$C_{1j} = \frac{b_j}{2\beta_j}\left[\frac{N_j + \beta_j}{N_j + 1}\right]$$

$$C_{2j} = AS_j + N_j\, P_j$$

For a given value of N_j the optimal T_j^* can be obtained as follows:

$$\frac{\partial TC_j}{\partial T_j} = 0 \qquad \Rightarrow - C_{2j} + C_{1j} T_j^{*2} = 0$$

$$T_j^* = [C_{2j}/C_{1j}]^{1/2}$$

Substituting this back, we get $TC_j(N_j, T^* (N_j))$ as:

$$TC_j = 2 T_j^* . C_{1j} + a_j$$

$$= 2 [C_{2j} . C_{1j}]^{1/2} + a_j$$

To obtain some idea about the probable range of the optimal value of N_j, we will treat N_j as a continuous variable. The total cost expression for equipment j can be expressed in terms of N_j explicitely as:

$$TC_j = \frac{A_j + N_j P_j}{T_j} + a_j + \frac{b_j}{2\beta_j} . T_j . \frac{\beta_j + N_j}{N_j + 1}$$

Now minimizing this with respect to T_j and N_j, we get,

$$\frac{\partial TC_j}{\partial T_j} = 0 \qquad T_j^* = \left[\frac{A_j + N_j P_j}{\frac{b_j}{2\beta_j} \left(\frac{\beta_j + N_j}{N_j + 1}\right)} \right]^{1/2}$$

$$\frac{\partial TC_j}{\partial N_j} = 0 \qquad (N_j^* + 1) = \left[\frac{b_j}{2\beta_j . P_j} (\beta_j - 1) \right]^{1/2} . T_j$$

$$= \left[(\beta_j - 1)\left(\frac{A_j}{P_j} - 1\right) \right]^{1/2}$$

Equation (2.8) gives some idea about the optimal value of N_j. This can be used as an initial guess for the value of N_j^*. Now if we do not permit N_j^* to be non-integer then it can be seen easily that optimality requires,

$$TC_j\,[N_j^*-1,\ T_j\,(N_j^*-1)] \geq TC_j\,[N_j^*,\ T_j\,(N_j^*)]$$

$$TC_j\,[N_j^*+1,\ T_j\,(N_j^*+1)] \geq TC_j\,[N_j^*,\ T_j\,(N_j^*)]$$

above conditions can finally be reduced to:

$$Z_j\,(N_j^*) \leq 1$$

$$Z_j\,(N_j^*+1) \geq 1 \qquad (2.9)$$

where,

$$Z_j(N_j) = \frac{AS_j + N_j\,P_j}{AS_j + (N_j-1)P_j}\left[\frac{N_j}{N_j+1}\right]\left[\frac{\beta_j + N_j}{\beta_j+N_j-1}\right]$$

Steps for obtaining the optimal solution can now be written as:

1. Obtain the initial guess for N_j^* from Eqn. (2.8)
2. If $N_j^* = 0$ then stop, otherwise vary N_j in the vicinity of the value obtained in first step till condition (2.9) is satisfied, Stop.

CHAPTER III

MIXED AND JOINT REPAIR POLICIES

3.1 Introduction:

In this chapter we will consider mathematical formulation, and solution procedure for obtaining optimal repair schedule for a multi equipment system for which there exists a positive common component of major repair cost. This common component can be shared by two or more equipments when they are repaired together. It may turn out to be profitable, then, to schedule major repairs for two or more equipments simultaneously. We will call this general problem as mixed repair scheduling problem because the common major repair cost is shared by more than one equipments when their major repairs are scheduled under the same common set-up. It can also be noted that the problem considered in Chapter II is a special case of this general problem, with the common component of major repair cost being equal to zero. Here also, it is assumed that the number of minor and major repairs per unit time for each equipment is independent of the production cost function and repair parameters of other equipments.

Here again the objective is to minimize total cost per unit time. Total cost comprises of production cost, repair cost and the downtime cost for the sake of simplicity of the mathematical formulation, we will make an additional assumption that whenever any major repair takes place some equipment, say r, must necessarily be repaired. Our decision variables will now be the time interval between two consecutive major repairs of this basic equipment r (basic time period), the number of major repairs of equipment r for each major repair of equipment j (m_j), and the number of minor repairs for each equipment between two consecutive major repairs of that equipment (N_j).

In Section 3.2, we shall introduce some additional assumptions and notations which will be used in developing the mathematical formulation for the mixed repair scheduling problem. We will, then, develop the mathematical formulation and solution procedure for this mixed repair scheduling problem in Section 3.3. Some special cases of this with negligible repair durations and linear production cost function have been considered in Section 3.4. To illustrate the solution procedures for one general mixed repair scheduling problem we take a numerical example in Section 3.5. As defined in Chapter I, when all the equipments are forced to have their major repairs simultaneously, the corresponding repair policy is known as joint repair policy. Mathematical

formulation and solution procedures for general joint repair scheduling problem along with a special case and a numerical example will be presented in Section 3.6. In Section 3.8, we have considered some special systems which have mathematical formulations parallel to what we have for our present mixed repair scheduling problem.

3.2 Assumptions and Notations:

3.2.1 Assumptions:

In addition to the assumptions made in sub-section 2.2.1 of Chapter II, we make the following assumptions:

9) The time interval between two consecutive major repair for any event is an integer multiple of the basic time period for the system.

In Chapter I we have defined basic time period for a system as the time interval between two consecutive major repairs of equipment r where equipment r is that equipment which has most frequent major repairs.

10) Any major repair can take place only when the equipment r is having its major repair.

11) Exponent of production cost function is the same for all equipments.

3.2.2 Notations:

In addition to the notation introduced in section 2.2.2

of Chapter II we introduce two more notations:

- A Common component of the major repair cost
- m_j Number of major repairs of basic equipment r for each major repair of equipment j ($j = 1 \ldots M$, $j \neq r$ and $m_r = 1$).

3.3 Problem Formulation and Solution Procedure:

3.3.1 Problem Formulation:

Figure 3.1 depicts the sequence of events for a system of 3 equipments ($M = 3$). From Figure 3.1 relationship between t_j and T can be written as,

$$m_j T = (N_j + 1)\, t_j - N_j\, d_j - d_M$$

$$t_j = \frac{m_j T - N_j d_j - d_M}{(N_j + 1)}$$

Various components of the total cost expression can be obtained as given below:

$$\text{Repair cost per basic cycle} = A + \sum_{j=1}^{M} \frac{AS_j}{m_j} + \sum_{j=1}^{M} \frac{P_j N_j}{m_j}$$

$$\text{Downtime cost per basic cycle} = \sum_{j=1}^{M} \frac{N_j d_j + d_M}{m_j} \cdot R_j$$

The production cost function for a equipment j, in terms of its effective age is given as,

$$f_j(t) = a_j + b_j\, te_j^{\,n}$$

Total production cost per basic cycle for the system can be written as:

$$PC = \sum_{j=1}^{M} \frac{1}{m_j} \sum_{i=1}^{N_j+1} \int_{(i-1)\frac{t_j}{\beta_j}}^{(i-1)\frac{t_j}{\beta_j} + t_j} (a_j + b_j t^n)\, dt$$

$$= \sum_{j=1}^{M} [\frac{1}{m_j} \{a_j (N_j+1) t_j + \frac{b_j t_j^{n+1}}{n+1} \cdot \sum_{i=0}^{N_j} [(\frac{i}{\beta_j} + 1)^{n+1} - (\frac{i}{\beta_j})^{n+1}]\}]$$

Now substituting t_j in terms of T, we get:

$$PC = \sum_{j=1}^{M} [\frac{1}{m_j} \{a_j (m_j T - N_j d_j - d_M) + b_j \frac{1}{n+1} \cdot (\frac{m_j T - N_j d_j - d_M}{N_j+1})^{n+1} \sum_{i=0}^{N_j} [(\frac{i}{\beta_j} + 1)^{n+1} - (\frac{i}{\beta_j})^{n+1}]\}]$$

Let us now define few new variables as:

$$C_{1j} = \frac{b_j}{n+1} [\frac{1}{N_j+1}]^{n+1} \sum_{i=0}^{N_j} [(\frac{i}{\beta_j} + 1)^{n+1} - (\frac{i}{\beta_j})^{n+1}]$$

$$C_{2j} = AS_j + P_j N_j + (N_j \cdot d_j + d_M)(R_j - a_j) \qquad (3.1)$$

$$C_2 = A + \sum_{j=1}^{M} \frac{C_{2j}}{m_j}$$

$$DN_j = d_j N_j + d_M$$

Using these new variables we can rewrite our total cost per unit time for the system as:

$$TC = \frac{C_2}{T} + \sum_{j=1}^{M} [a_j + \frac{C_{1j}}{T} \frac{(n_j T - N_j d_j - d_M)^{n+1}}{m_j}] \quad (3.2)$$

3.3.2 Solution Procedure:

The optimal repair schedule for our system can be obtained by unconstrained optimization of the total cost function. Out of the total (2M+1) decision variables (2M are the numbers of N_j' s and M_j' s, one for each equipment and one is the basic time period T), 2M are integer decision variables and one is continuous. Study of equation 3.2 shows that even though our objective function is unimodular in T for a given set of N_j' s and m_j' s, nothing can be said about its unimodularity with respect to T and the optimal N_j' s and m_j' s for this T (N_j' (T) and m_j (T)). It can also be noted that C_{2j}, C_{1j} and DN_j are functions of N_j alone. We suggest three solution procedures for obtaining the optimal solution of the mixed repair scheduling problem. Procedure I and II are on the lines parallel to the solution procedures given in Chapter II.

(1) Procedure I (Exhaustive Search Method):

As the name suggests, to obtain the optimal solution for the system we try all possible combinations of all the integer decision variables and analytically obtain the optimal

value of the basic time period T for each such set of integer decision variables. We also calculate the objective function value for each such combination. For a given set of N_j's and m_j's the optimal value of basic time period can be obtained by setting the partial derivative of the total cost function with respect to T, equal to zero and solving the resulting equation.

$$\frac{\partial TC}{\partial T} = 0$$

$$-\frac{A}{T^{*2}} - \frac{1}{T^{*2}}\sum_{j=1}^{M}\frac{C_{2j}}{m_j} + \sum_{j=1}^{M}\frac{(m_jT^* - DN_j)^n\,(n.m_jT^* + DN_j)}{T^{*2}} = 0$$

It can be seen that the solution of this equation corresponds to the minimum value of the objective function because for any set of N_j's and m_j's $\partial^2TC/\partial T^2$ is always positive. Using some iterative techniques we can find out the optimal T^* for this given set of N_j's and m_j's. Substituting this back in the expression for the objective function we get:

$$TC = a_j + \sum_{j=1}^{M} C_{1j}\,(m_jT^* - DN_j)^n.(n+1)$$

The solution corresponding to that set of decision variables, for which the value of objective function is minimum, is selected as optimal solution to the problem.

This procedure can be stated in following steps:

Step 1: Select a set of N_j's.

Step 2: Calculate the values of C_{ij}'s, C_{2j}'s and DN_j's.

<u>Step 3</u>: Select a set of m_j' s.

<u>Step 4</u>: Obtain the basic time period corresponding to present set of N_j' s and m_j' s. Calculate the total cost function value corresponding to this value of T.

<u>Step 5</u>: Repeat step 4 for various values of m_j set.

<u>Step 6</u>: Repeat from Step 2 to Step 5 for various sets of N_j' s.

<u>Step 7</u>: Select that combination of N_j' s, m_j' s and $T^*(N_j, m_j)$ which corresponds to minimum cost.

The possible combinations of N_j' s and m_j' s, which are to be tried, are decided empirically.

(2) <u>Procedure II</u>:

Similar to the procedure discussed in Chapter II, here also we will make a continuous search along the continuous variable T, and by doing so we will divide the entire range (feasible and empirically selected) of T in several intervals. The optimal values of N_j' s and m_j' s remain same for every point within each such interval. Since the values of $N_j^*(T)$ and $m_j^*(T)$ remain constant for all the values of T within an interval, our total cost function becomes unimodular in T and $N_j^*(T)$ and $m_j^*(T)$ in that interval. As we have already found out the optimal values of N_j' s and m_j' s for each such interval, while dividing the entire range of T in above mentioned intervals, all what we require to do, now,

is to obtain the optimal T and TC for the corresponding N_j^{*}'s and m_j^{*}'s within each interval. The global optimal can be obtained by selecting the best out of all these local optimum solutions. The procedure can be understood from the flowchart drawn in Figure 3.2. The related variables are explained in Table 3.1.

Table 3.1: Notations used in Flow Chart: Fig.3.2.

T_L	Lower (empirical) limit on T
T_U	Upper (empirical) limit on T
T_{L_i}	Lower limit of i-th interval
T_{U_i}	Upper limit of i-th interval
δ	Step length (initial)
N_{N_j}	New optimal N_j
m_{N_j}	New optimal m_j
i_{max}	Maximum limit on number of intervals.
K_{max}	K_{max} along with δ determines the accuracy about determining the end limits of intervals.
T_{opt_i}	Optimal T in i-th interval.
TC_{opt_i}	Optimal value of total cost in i-th interval
TC_{u_i}	Value of total cost of upper limit of i-th interval.
TC_{L_i}	Value of total cost of lower limit of i-th interval.

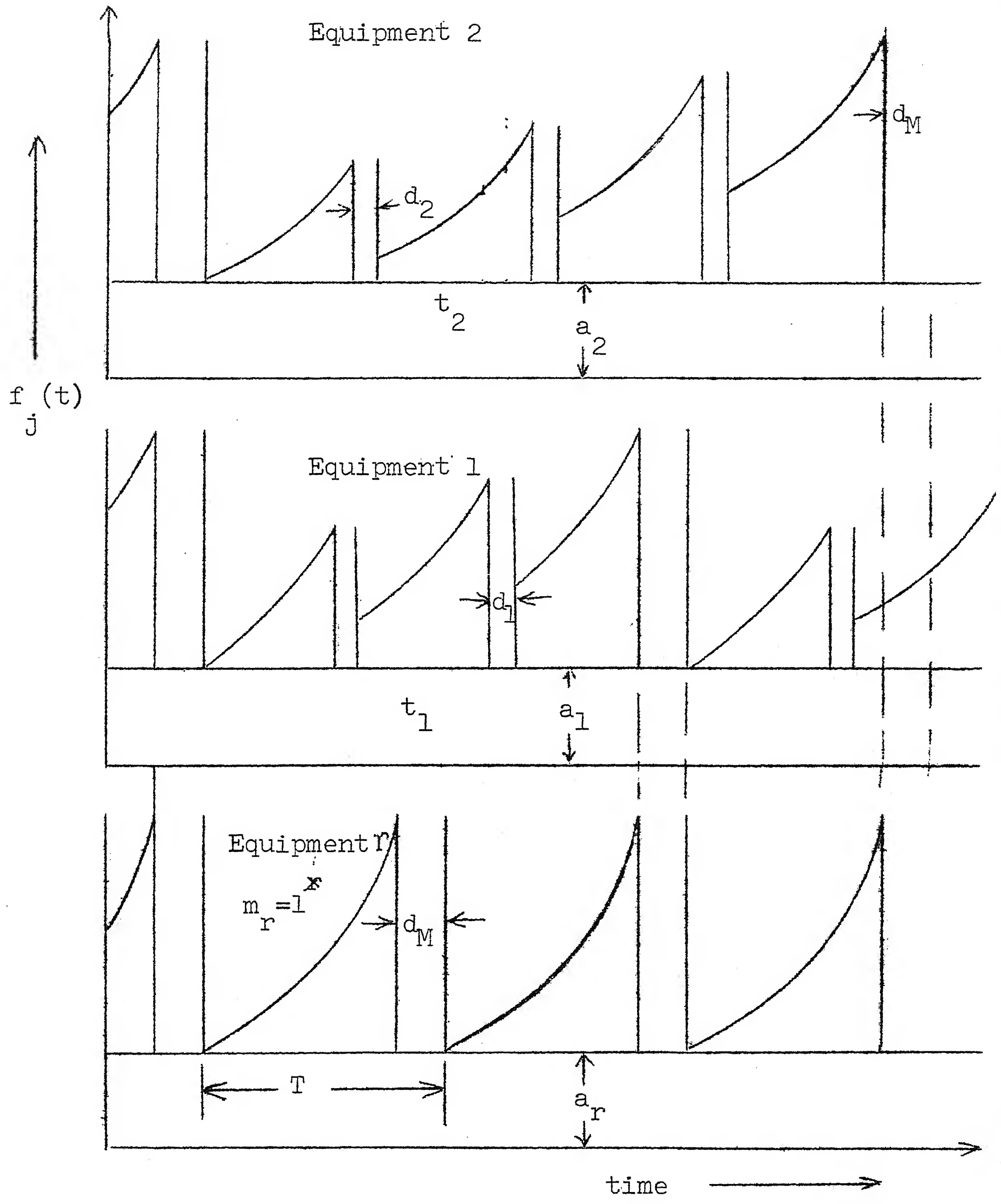

Fig. 3.1: Mixed repair scheduling for a goods producing system.

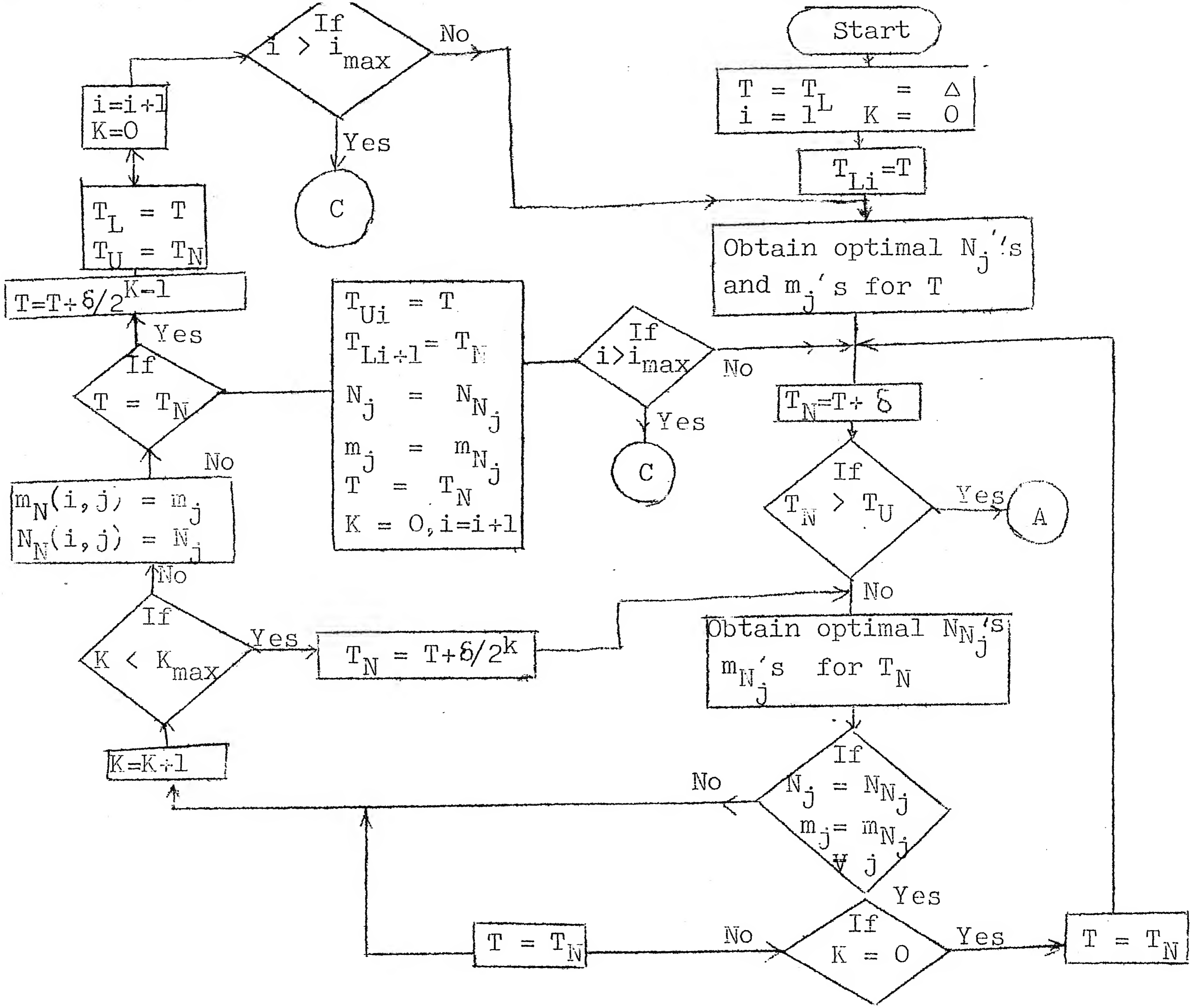

Fig. 3.2: (Procedure II, Step 1): Dividing the entire range of T in various intervals.

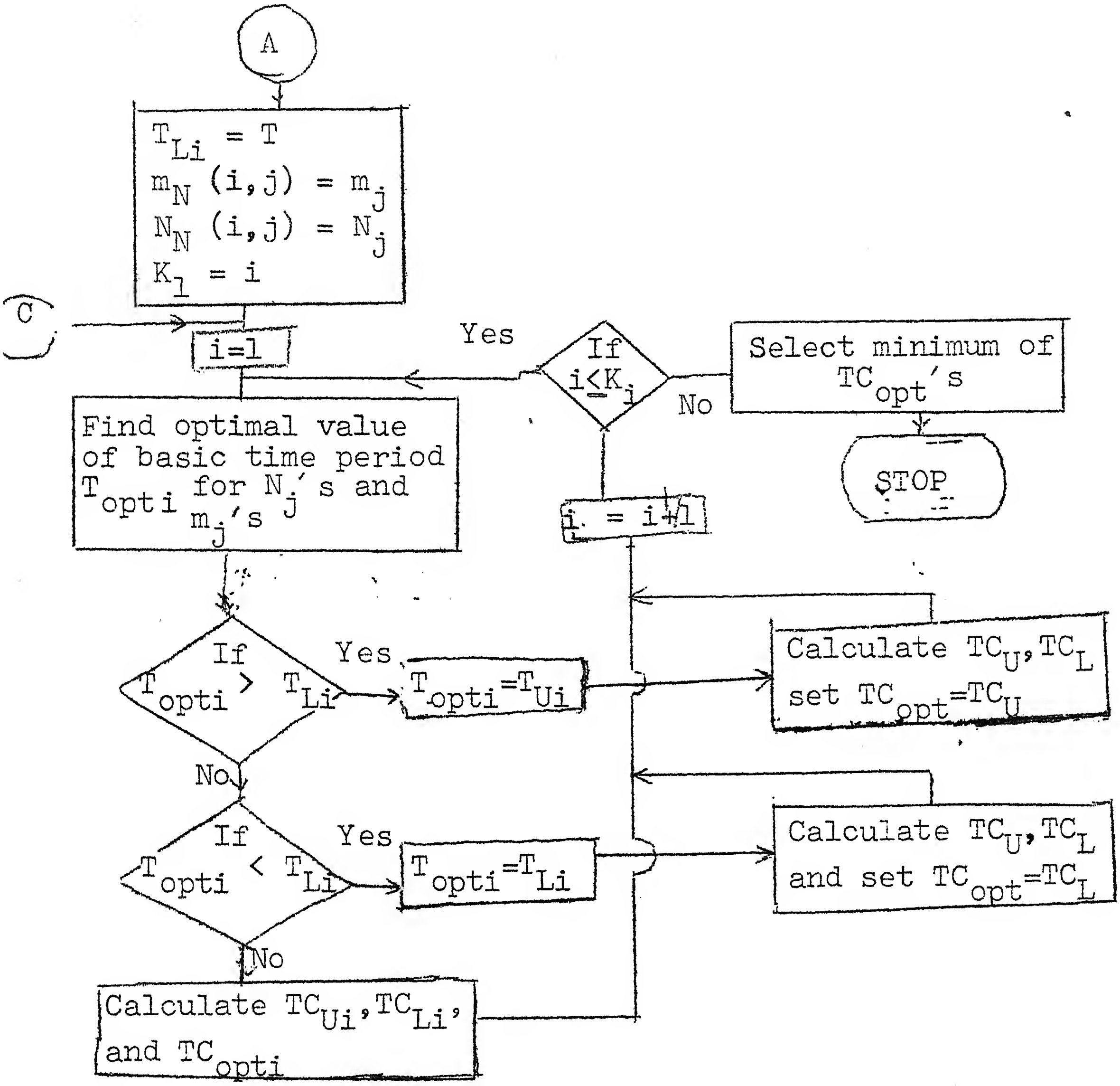

Fig. 3.2: (Continued). Procedure II (Step 2 and 3).

(Calculating the optimum T and TC for each interval and selecting the best set.)

Following steps are required to be followed for obtaining the optimal solution.

Step 1: (Dividing the feasible range of T)

To start with select an empirical feasible range for the optimum value of T. Start with the lowest value of T in this range. Keep varying the value of T and find the values of $N_j^*(T)$ and $m_j^*(T)$ for each value of T. That range of T within which $N_j^*(T)$ and $m_j^*(T)$ do not vary can be considered as one subrange or interval. Thus the entire range of T is divided in several intervals.

Step 2: For each such interval find out the optimum value of T^* and calculate the total cost for this T^* and corresping N_j^*'s and $m_j^{*'}$s.

Step 3: Select the best $N_j^{*'}$s and $m_j^{*'}$s and the corresponding T^* which gives lowest value of the objective function. This corresponds to the global optimal value of the objective function.

(3) Procedure III:

In procedure II we did not make use of the fact that the total cost, corresponding to the local minima of each of the intervals mentioned above, is unimodular in T . In present procedure we will utilize this property of our problem for obtaining the optimal solution. Here we make a modified block

search along the continuous variable T to obtain the global optimum solution for our problem. This method is explained in the flow chart given in Figure 3.3. This procedure can be stated in following steps:

Step 1: Select A and B as the two extreme values, that T can lie in. This selection is totally empirical, based on guess or experience.

Step 2: Obtain the four initial points as,

$$T_1 = A$$
$$T_4 = B$$
$$T_3 = A + (F_N/F_{N+1})\ (B-A)$$
$$T_2 = T_1 + (T_4 - T_3)$$

Here F_i is the i-th term in standard Febonacci series and N can be selected empirically.

Step 3: (a) Set $i = 1$

(b) Obtain the optimal values of N_j' s and m_j' s for T_i. Obtain T_i^* for this set of N_j ' s and m_j ' s. If this $T_i^*(N_j$'s, m_j ' s) is different from T_i then set $T_i = T_i^*$. Obtain the value of the objective function for this set of decision variables. Denote this as TC_i.

(c) Repeat Step (3.b) for i = 2, 3 and 4.

Step 4: If N_j^{*}' s and m_j^{*}' s for T_2 and T_3 are same then go to Step 7, otherwise go to Step 5.

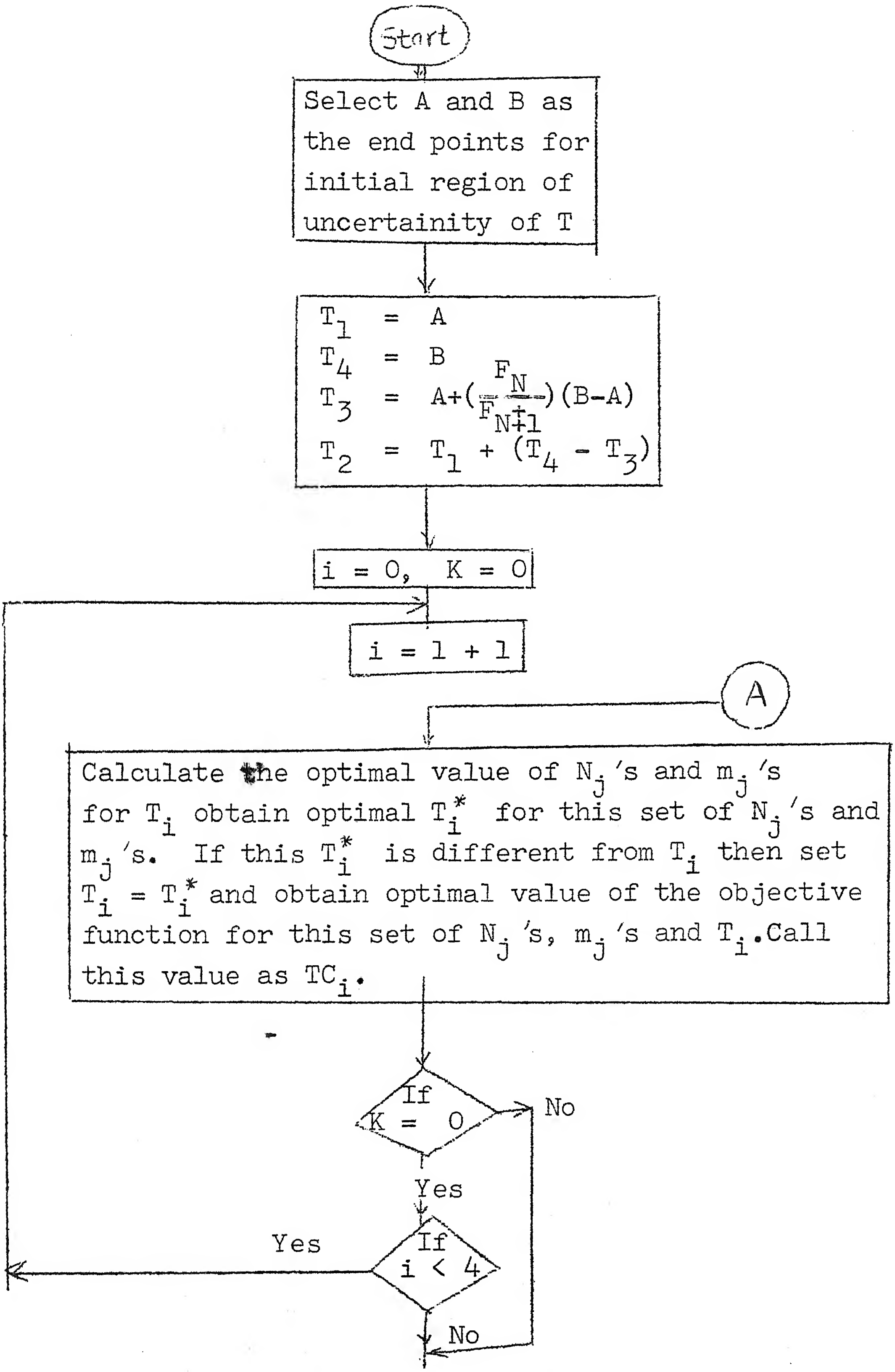

Fig. 3.3: Flow chart for procedure 3.

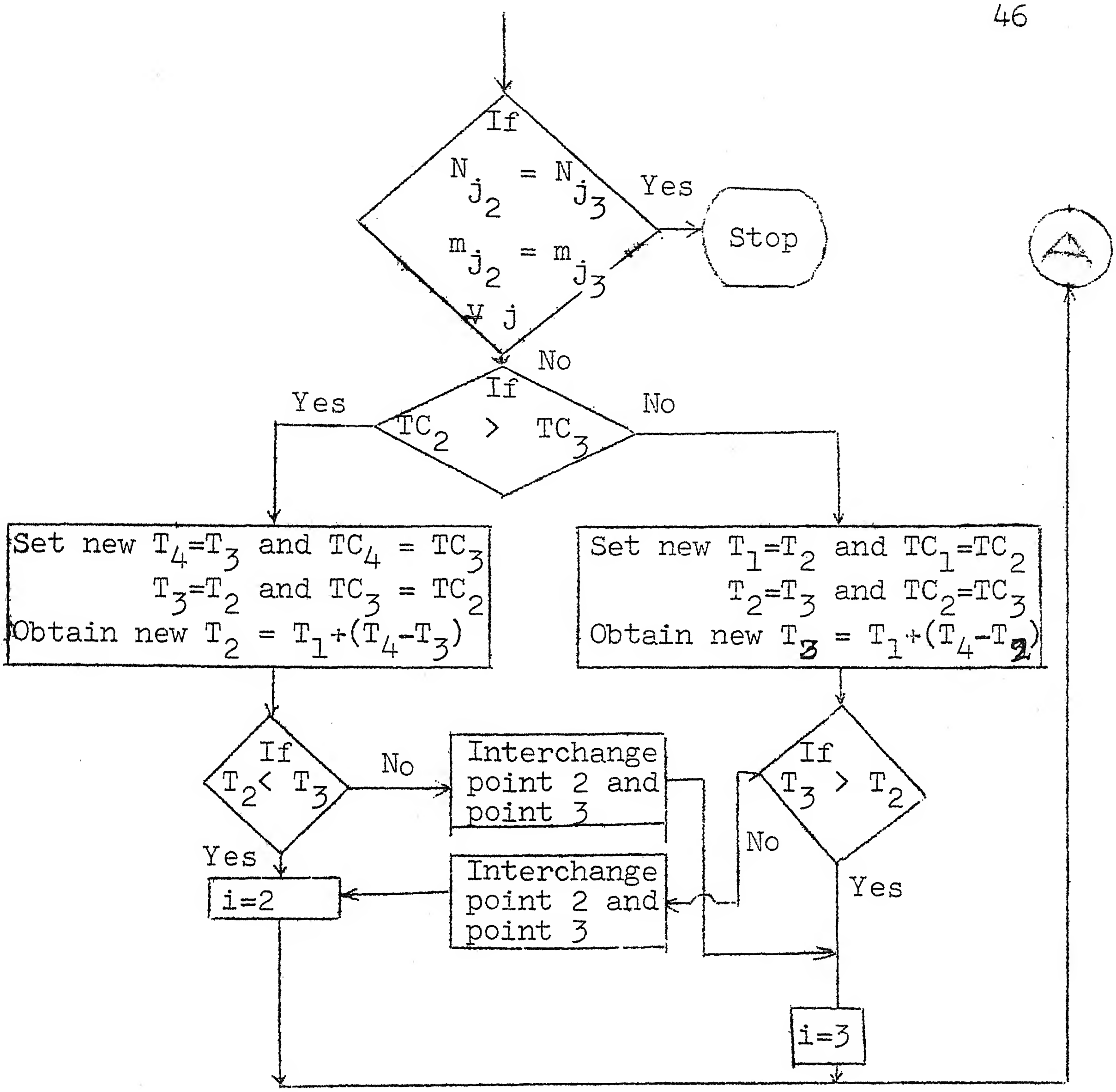

Fig. 3.3 (Continued): Flow chart for procedure 3.

Step 5: Select the smaller of TC_2 and TC_3.

If TC_3 is smaller then

set new $T_1 = T_2$ and $TC_1 = TC_2$

$T_2 = T_3$ and $TC_2 = TC_3$

obtain new $T_3 = (T_4 - (T_2 - T_1))$

If T_3 is greater than T_2 then set $i = 3$ and go to Step 6 otherwise interchange point 2 and point 3, set $i = 2$ and go to Step 6.

If TC_2 is smaller then

set new $T_4 = T_3$ and $TC_4 = TC_3$

$T_3 = T_2$ and $TC_2 = TC_3$

obtain new $T_2 = (T_1 + (T_4 - T_3))$

If T_2 is smaller than T_3 then set $i = 2$ and go to Step 6 otherwise interchange point 3 and point 2, set $i = 3$ and go to Step 6.

Step 6: Obtain the optimal value of N_j' s and m_j' s for T_i. Obtain T_i^* for this set of N_j' s and m_j' s. If this $T_i^*(N_j's, m_j's)$ is different from T_i then set $T_i = T_i^*$. Obtain the value of the optimal objective function for this set of decision variables. Denote this as TC_i. Go to Step 4.

Step 7: Both point 2 and point 3 correspond to the same solution therefore we can select any one as our required solution.

Note: Procedure III is a heuristic and there are some chances that it may miss the optimum solution but in most of several examples that we have tried, it was found that procedure III and procedure II give exactly same result. In general C.P.U. time for procedure III is nearly one tenth to one twentieth of that for procedure II. It can be seen that even for small number of equipments, procedure one requires very large number of iterations because the total number of combinations of N_j's and m_j's is usually very large. For a system with three components only, even if we have some guess about the optimal solution and accordingly try to restrict our exhaustive search for every integer variable to only very few points (say five) then the number of total iterations required for optaining the optimal solution is 5^6. This obviously is a very large number and therefore it is very uneconomical to use procedure I. It is preferable to go for either procedure II or procedure III depending upon the computational efforts and the desirability of the accuracy of optimal solution, so obtained. Procedure III with considerably less calculations can yield a near optimum or sometimes optimum solution, whereas procedure II will give the optimal solution but the computational efforts will be large.

3.4 Numerical Example:

We consider a system of three equipments. All relevent informations about the system are given below:

Number of equipments = 3

Duration of a major repair = 0.6 time units

Table 3.2: Input data for numerical example.

Parameters	Equipment 1	Equipment 2	Equipment 3
Cost of one minor repair	30.0	20.0	10.0
Cost of one major repair	40.0	60.0	80.0
Downtime cost rate	60.0	100.0	140.0
Duration of minor repair	0.30	0.20	0.10
Improvement factor	5.0	4.0	3.0
Production cost function	$5+2t^2$	$5+4t^2$	$5+6t^2$

Common component of the major repair cost = 20.0

Solution:

We shall solve this problem using procedure II. To illustrate this procedure we first divide the entire range of T in several intervals.

We set $T_U = 12.0$, $T_L = 4.9$, $= 1.0$, $i_{max} = 50$ and $K_{max} = 5$. Here T_U and T_L are the upper and lower limits respectively of the initial interval of uncertainty. δ denotes initial step length. $\delta/2^{K_{max}}$ gives the accuracy with which we can determine the end points for each interval. i_{max} denotes maximum number of intervals which can be considered (an empirical limit).

We start $T = 4.9$

The optimal values of N_j s and m_j s are {2, 1, 1} and {2, 1, 1} respectively.

$T_N = T + \delta = 5.9$

The optimal values of N_{N_j}' s and m_{N_j}' s are {1, 1, 2) and {1, 1, 1} respectively.

Obviously the interval in which N_j' s and m_j' s are optimal ends before $T = 5.9$. We now make a search at $T_N = T + \delta/2 = 5.4$.

The optimal values of N_{N_j}' s and m_{N_j}' s are {1, 1, 2} and {1, 1, 1}. This again cannot come in the interval for which {2, 1, 1} and {2, 1, 1} are optimal. Our next new value of basic time period is now $T_N = T + \delta/2^2 = 5.150$. We keep repeating this process till our final increment becomes equal to $\delta/2^5$. Once the upper limit for interval one is obtained it automatically fixes the lower limit of interval two. Table 3.3 gives the values of optimal N_j' s and m_j' s for different values of basic time period.

Table 3.3: Obtaining first interval for numerical example.

T_{basic}	N_j^* s			m_j^* s		
4.9	2	1	1	2	1	1
5.9	1	1	2	1	1	1
5.4	1	1	2	1	1	1
5.15	2	1	22	2	1	1
5.025	2	1	2	2	1	1
4.963	2	1	2	2	1	1

Even at 4.063 the values of optimal N_j' s and m_j' s are different from that of the values of N_j^* s and m_j^*s at 4.9. Thus the upper limit for interval one is decided as 4.9 itself. Lower limit for next interval is set at 4.963 and the corresponding N_j^* s and m_j^* s are {2,1,2} and {2, 1, 1} respectively.

Similarly upper and lower limits for all the intervals are obtained. The limits of each intervals along with the optimal N_j's and m_j's are given in the Table 3.4. Optimum values of total cost are obtained for each interval. These values are given in the Table 3.5. T_j in Table 3.5, denotes the optimum basic operation duration for equipment j. The best solution out of all above solutions is selected. This optimum solution is given in Table 3.6.

T.BASIC FROM	T.BASIC TO	N(J)S N1	N2	N3	M(J)S M1	M2	M3
4.000	4.900	2	1	1	2	1	1
4.963	5.275	2	1	2	2	1	1
5.359	6.359	1	1	2	1	1	1
6.431	6.431	1	1	3	1	1	1
6.494	7.619	1	2	3	1	1	1
7.681	8.431	1	2	4	1	1	1
8.494	8.494	2	2	4	1	1	1
8.556	8.744	2	3	4	1	1	1
8.838	9.775	2	3	5	1	1	1
9.806	10.306	2	3	6	1	1	1
10.369	10.681	2	4	6	1	1	1
10.775	11.275	2	4	7	1	1	1
11.338	12.000	3	4	7	1	1	1

TABLE 3.5: LOCAL OPTIMAL SOLUTIONS FOR EACH INTERVALS.

N1	M1	T(1)	N2	M2	T(2)	N3	M3	T(3)	T.COST	TBASIC
2	2	2.867	1	1	2.050	1	1	2.100	133.65	4.900
2	2	3.117	1	1	2.238	2	1	1.492	131.83	5.275
2	2	3.438	1	1	2.253	2	1	1.502	131.72	5.306
1	1	2.673	1	1	2.723	2	1	1.815	128.50	6.246
1	1	2.766	1	1	2.816	3	1	1.383	128.56	6.431
1	1	3.091	2	1	2.029	3	1	1.546	127.48	7.083
1	1	3.391	2	1	2.227	4	1	1.336	128.34	7.681
2	1	2.431	2	1	2.498	4	1	1.499	130.93	8.494
2	1	2.452	3	1	1.832	4	1	1.511	131.05	8.656
2	1	2.525	3	1	1.894	4	1	1.555	131.43	8.775
2	1	2.546	3	1	1.909	5	1	1.290	131.53	8.838
2	1	2.869	3	1	2.152	6	1	1.229	134.47	9.806
2	1	3.056	4	1	1.794	6	1	1.310	137.04	10.369
2	1	3.171	4	1	1.863	6	1	1.359	138.56	10.713
2	1	3.192	4	1	1.875	7	1	1.184	138.95	10.775
3	1	2.459	4	1	1.988	7	1	1.255	141.95	11.338

TABLE 3.6: GLOBAL OPTIMAL SOLUTION.

THE OPTIMAL SOLUTION

N*(1) = 1 M*(1) = 1

N*(2) = 2 M*(2) = 1

N*(3) = 3 M*(3) = 1

TOTAL COST PER UNIT TIME = 127.48

OPTIMAL BASIC TIME PERIOD = 7.083

3.5 Special Case:

(Negligible repair durations and linear production cost function.)

$$d_M = d_j = 0 \qquad \text{for all } j$$

$$n = 1$$

When the repair durations for the system are very small or negligible compared to the operation durations then we can neglect this for the sake of simplification of our mathematical formulation. The downtime cost for these repairs will be included in the repair costs. If for such system, unit production cost also varies linearly with effective age, then total cost expression gets highly simplified.

The variables C_{1j}, C_{2j}, as defined in Section 3.3.1, can now be written as follows:

$$C_{1j} = \frac{b_j \, (N_j + \beta_j)}{2(N_j + 1) \, \beta_j}$$

$$C_{2j} = AS_j + P_j \, N_j$$

Expression for C_2 in terms of C_{2j}' s remains unchanged and DN_j is equal to zero. Proceeding in the similar manner, as we did in Section 3.3.1, we can write the expression for total cost per unit time as:

$$TC = \frac{C_2}{T} + \sum_{j=1}^{M} [a_j + C_{1j} . m_j \, T] \qquad (3.3)$$

Solution Procedure:

For a given set of N_j's and m_j's the optimum value of T is obtained by setting the partial derivative of total cost with respect to T equal to zero.

$$\frac{\partial TC}{\partial T} = 0$$

$$\Rightarrow \quad T^* = [C_2/(\sum_{j=1}^{M} C_{1j} \cdot m_j)]^{1/2} \qquad (3.4)$$

To obtain some initial guess about the values of m_j^*'s, we will treat these variables as continuous variables.

$$\frac{\partial TC}{\partial m_j} = 0$$

$$\Rightarrow \quad m_j^* = 1/T \, [C_{2j}/C_{1j}]^{1/2} \qquad (3.5)$$

The expression for m_j in terms of m_k (where k is some other equipment) can be written as:

$$m_j^* = (K_{N_j}/K_{N_k}) \cdot m_k^* \qquad (3.6)$$

where, $$K_{N_j} = (C_{2j}/C_{1j})^{1/2}$$

Similarly if N_j's are also treated as continuous variable then some initial guess about the optimum N_j's can be obtained as given below.

$$\frac{\partial TC}{\partial N_j} = 0$$

$$\Rightarrow (N_j^* + 1) = [(\beta_j - 1)(\frac{AS_j}{P_j} - 1)]^{1/2} \quad \text{for } \beta_j < \frac{AS_j}{P_j}(\beta_j - 1)$$

$$= 1 \qquad \text{for } \beta_j \geq \frac{AS_j}{P_j}(\beta_j - 1)$$

The solution procedure for obtaining the optimal solution can be stated in following steps.

Step 1: Obtain N_j^*s from equation (3. 7) for all the equipments.

Step 2: Obtain C_{1j}, C_{2j}, C_2 and K_{N_j} for all the equipments. Select that equipment which has minimum K_{N_j} (say equipment r) and set $m_r = 1$. Obtain approximate integer values of m_j^* s for all the equipments from equation (3. 5).

Obtain the optimal basic time period from Eq.(3.4) and the corresponding total cost per unit time from (3. 3). This solution is an approximate optimal solution.

Note: Since we have treated both N_j' s and m_j's are continuous variables the solution obtained from above procedure may not be optimal solution. To obtain the exact solution, we can try all the combinations of m_j's and N_j's which are in the neighbourhood of N_j^* s and m_j^* s.

3.6 Joint Repair Scheduling Problem:

3.6.1 Introduction:

For those goods producing systems, where the common set up cost for major repairs is very large, it will, in most of the cases, turn out to be economical to force all the equipments to have their major repairs simultaneously. By

performing these major repairs, simultaneously we can save on the set up costs of repair for each individual equipment.

3.6.2 Assumptions:

In addition to the assumptions of Section 3.2.1, we will make one more assumption.

12) All the equipments have to undergo major repairs simultaneously; that is, value of m_j for each equipment is one.

3.6.3 Problem Formulation and Solution Procedure:

3.6.3.1 Problem Formulation:

Figure 3.4 shows the sequence of events for joint repair policy of a goods producing system. From Fig. 3.4, relationship between t_j and T can be obtained as:

$$t_j = \frac{T - N_j d_j - d_M}{(N_j + 1)}$$

$$\text{Repair cost per cycle} = A + \sum_{j=1}^{M} [AS_j + P_j N_j]$$

$$\text{Downtime cost per cycle} = \sum_{j=1}^{M} (N_j d_j + d_M) R_j$$

Production cost per cycle for the system can be written as:

$$PC = \sum_{j=1}^{M} [a_j (T-N_j d_j - d_M) + \frac{b_j}{n+1} (\frac{T-N_j d_j - d_M}{N_j+1})^{n+1} . \sum_{i=0}^{N_j} [(\frac{i}{\beta_j} + 1)^{n+1} - (\frac{i}{\beta_j})^{n+1}]]$$

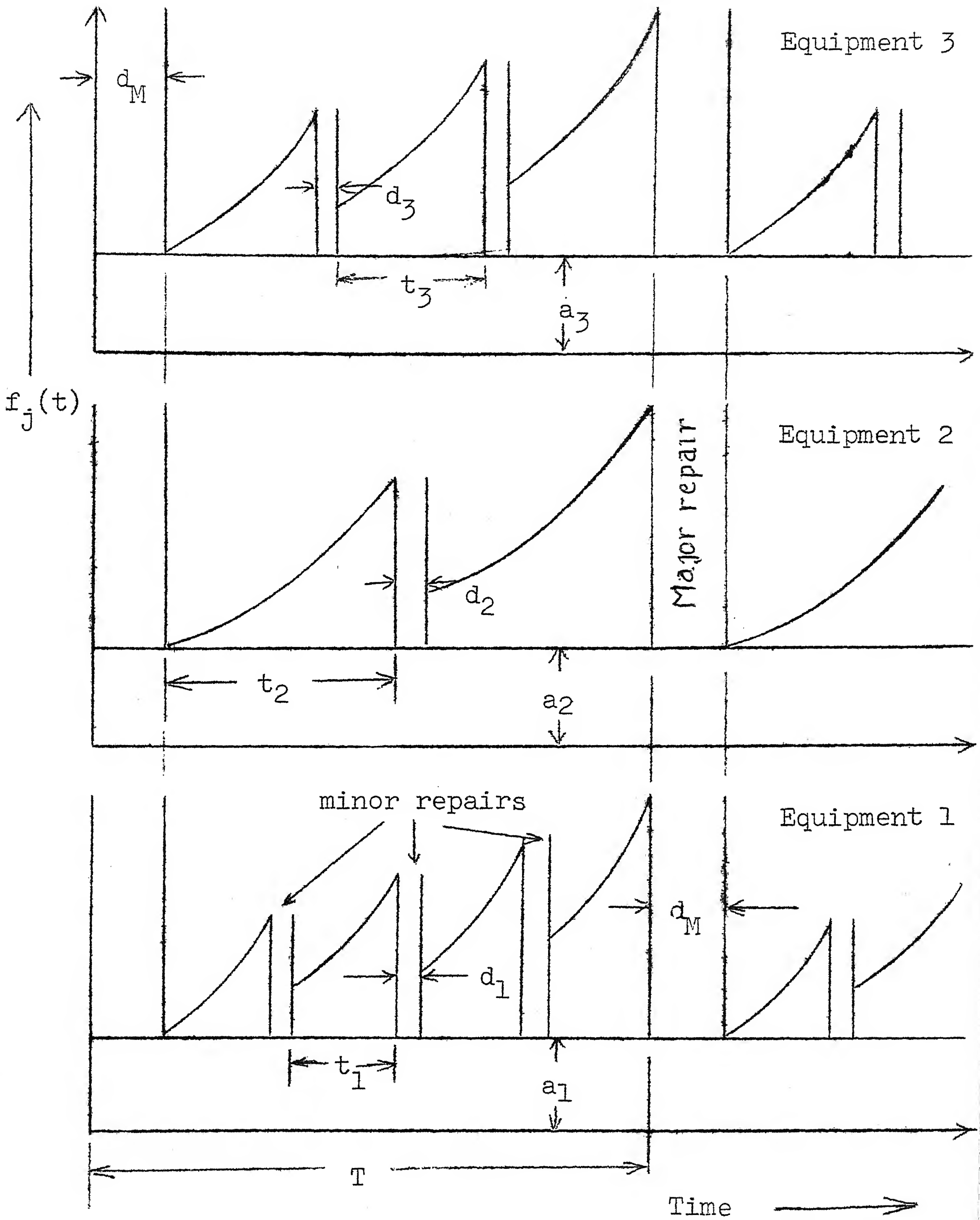

Fig. 3.4: Joint repair scheduling for a goods producing systems.

Our variables DN_j, C_{1j}, C_{2j} and C_2 can be written as:

$$DN_j = d_j N_j + d_M$$

$$C_{1j} = \frac{b_j}{n+1} \left(\frac{1}{N_j+1}\right)^{n+1} \sum_{i=0}^{N_j} \left[\left(\frac{i}{\beta_j} + 1\right)^{n+1} - \left(\frac{i}{\beta_j}\right)^{n+1}\right]$$

$$C_{2j} = AS_j + P_j N_j + DN_j (R_j - a_j)$$

$$C_2 = A + \sum_{j=1}^{M} C_{2j}$$

The total cost expression for the system becomes:

$$TC = \frac{C_2}{T} + \sum_{j=1}^{M} \left[a_j + \frac{C_{1j}}{T} (T - DN_j)^{n+1}\right]$$

3.6.3.2 Solution Procedure:

Unconstrained optimization of the total cost function, TC will give the optimal repair schedule for the problem. Total number of decision variables are (M + 1). Out of these M decision variables (N_j's) are integers. Procedures I, II and III mentioned in Sec. 3.3.2, can be used for joint repair scheduling problem, with slight modifications in them.

1. Procedure I: (Exhaustive Search Method)

For a given set of N_j's optimal value of T can be obtained from the following equation,

$$\frac{\partial TC}{\partial T} = 0 \Rightarrow - C_2 + \sum_{j-1}^{M} C_{1j} [T^* - DN_j]^n [n.T^* - DN_j] = 0 \qquad (3.10)$$

Substituting this in expression for total cost we get:

$$TC = \sum_{j=1}^{M} a_j + \sum_{j=1}^{M} C_{1j} [T^* - DN_j]^n (n+1) \qquad (3.11)$$

The steps for obtaining the optimal solution can be written

Step 1: Select a set of N_j's.

Step 2: Calculate the values of C_{1j}'s, C_{2j}'s, C_2 and DN_j's.

Step 3: Obtain the basic time period for this set of N_j's, from equation (3.10). Calculate the total cost function value corresponding to this set of N_j's and T.

Step 4: Repeat Step 2 and Step 3 for all the possible combination of N_j's.

Step 5: Select that set of N_j's and the corresponding T^* which gives minimum total cost per unit time. This solution is optimum solution.

All the possible combinations of N_j's which are to be tried are decided empirically.

2. Procedure II:

In this procedure we divide the entire range of the continuous variable T in several intervals such that within each interval the total cost function is unimodular in T and corresponding $N_j^*(T)$. Where $N_j^*(T)$ is the optimal value of N_j for given T. The modified form of solution procedure II is presented in following steps.

Step 1: (Dividing the empirically selected feasible range of T) To start with we set T equal to the lower limit of the empirically selected feasible range of T. We keep varying the value of T in a particular fashion (refer to Fig. 3.2) and obtain the values $N_j^{*\prime}$s for each such value of T. That range of T, for which $N_j^{*}(T)'$ does not vary, can be considered as one subrange or interval. The entire range is thus divided in several intervals.

Step 2: For each such interval find out the optimum value of T and calculate the total cost for this T and the corresponding N_j's.

Step 3: Select that set of N_j's and corresponding T for which the total cost per unit time is minimum. This corresponds to the global optimal value of the objective function.

(3) Procedure III:

Since the local minima of each of the intervals, obtained in procedure two are such that the corresponding value of the total cost function is unimodular in T, we make use of modified block search procedure to obtain approximate optimal solution of the problem. The steps for this procedure are nearly same as that of procedure III in Sec. 3.3.2.

Step 1: Same as in Sec. 3.3.2.

Step 2: Same as in Sec. 3.3.2.

Step 3(a): Same as in Sec. 3.3.2.

Step 3(b): Obtain the optimal value of N_j's for T_i and obtain T_i^* for this set of N_j's. If $T_i^*(N_j\text{'s})$ is different from T_i then set $T_i = T_i^*(N_j\text{'s})$. Obtain the value of objective function for this set of decision variables and denote it as TC_i.

Step 3(c): Repeat Step 3(b) for i = 2, 3 and 4.

Step 4: If N_j^*s for T_2 and N_j^*s for T_3 are same then go Step 7 otherwise go to Step 5.

Step 5: Same as in Section 3.3.2.

Step 6: Obtain the optimal value of N_j's for T_i. Obtain T_i^* for this set of N_j's. If this T_i^* is different from T_i then set $T_i = T_i^*$. Obtain the objective function value for this set of N_j's and T_i and denote it as TC_i. Go to Step 4.

Step 7: Both point 2 and point 3 correspond to the approximate optimal solution.
Stop.

Note: The computational efforts for procedure III are minimum, but, then the solution obtained from this need not be the optimal one. In general when there is no idea about the optimal values of N_j's then the computational efforts for procedure II are much less than that of procedure I.

Procedure II is preferred under such circumstances. On the other hand, if some idea about the optimal N_j's can be obtained from some-where then computational efforts for procedure I get reduced enormously because now our exhaustive search is made **within** a limited region only. Procedure I is preferred to procedure II for such situations.

3.6.4 Numerical Example:

To illustrate procedure II we will take up an example: Let us consider the production system which we considered for mixed repair scheduling problem.

Solution: The empirical upper and lower limit on the initial interval of uncertainity, denoted by t_U and t_L, can be selected as 1.9 and 12.0 respectively. We also select $\delta = 1.0$, $K_{max} = 5$ and $i_{max} = 50$ (the terms have their usual meaning).

Following exactly the same procedure as we did in case of mixed repair scheduling problem, first interval can be obtained from the following table.

Table 3.7: Obtaining First Interval

T	N_j^* s		
	N_1	N_2	N_3
1.900	0	0	0
2.900	0	0	0
3.900	0	0	1
3.400	0	0	1
3.150	0	0	0
3.275	0	0	1
3.213	0	0	0
3.244	0	0	0

From the Table 3.7, it can be seen that the interval, in which {0, 0, 0} remains as the optimal set of values for N_j's sprawls upto T = 3.244. Thus,

$$T_{L_1} = 1.9 \quad \text{and} \quad T_{U_1} = 3.244$$

Similarly we find out the upper and lower limits for other intervals also. These limits along with the corresponding optimal value of N_j's is given in Table 3.8. The total number of such intervals are 15. In Table 3.9, we have calculated the optimal value of the objective function for each such interval. Values of operation durations for each equipments are also printed in that table. The optimal set of decision variables is, then, selected out of all these local optimal solutions.

The global optimum solution for the problem is given in Table 3.10.

3.6.5 Special Case:

(Linear production cost function with negligible repair durations).

$$d_M = d_j = 0 \qquad \text{for all } j \quad \text{and } n = 1.$$

The variables C_{1j}, C_{2j}, as defined in Sec. 3.3.1, can be written as following:

$$C_{1j} = \frac{b_j \, (N_j + \beta_j)}{2(N_j + 1) \, \beta_j}$$

$$C_{2j} = AS_j + P_j \, N_j$$

TABLE 3.8: LIMITS OF INTERVALS ALONG WITH CORRESPONDING OPTIMAL SET OF N(J)'S

T.BASIC FROM	T.BASIC TO	N(J)S N(1)	N(2)	N(3)
1.000	3.213	0	0	0
3.305	4.056	0	0	1
4.119	4.931	0	1	1
4.963	5.150	0	1	2
5.244	6.369	1	1	2
5.431	6.431	1	1	3
5.494	7.619	1	2	3
7.681	8.431	1	2	4
8.494	8.494	2	2	4
8.555	8.744	2	3	4
8.838	9.775	2	3	5
9.805	10.305	2	3	6
10.369	10.681	2	4	6
10.775	11.275	2	4	7
11.338	12.000	3	4	7

TABLE 3.9: LOCAL OPTIMAL SOLUTIONS FOR EACH INTERVAL.

N1	T(1)	N2	T(2)	N3	T(3)	TCOPT	TOPT
0	2.613	0	2.613	0	2.613	152.69	3.213
0	3.456	0	3.455	1	1.678	140.38	4.056
0	4.331	1	2.066	1	2.116	133.70	4.931
0	4.550	1	2.175	2	1.450	132.83	5.150
1	2.673	1	2.723	2	1.815	128.60	5.246
1	2.766	1	2.816	3	1.383	128.56	6.431
1	3.091	2	2.028	3	1.546	127.48	7.083
1	3.391	2	2.227	4	1.336	128.34	7.681
2	2.431	2	2.498	4	1.499	130.93	8.494
2	2.452	3	1.839	4	1.511	131.05	8.556
2	2.546	3	1.909	5	1.290	131.53	8.838
2	2.859	3	2.152	6	1.229	134.47	9.805
2	3.056	4	1.794	6	1.310	137.04	10.369
2	3.192	4	1.875	7	1.184	138.95	10.775
3	2.459	4	1.988	7	1.255	141.95	11.338

TABLE 3.10: GLOBAL OPTIMAL SOLUTION.

THE OPTIMAL SOLUTION

N*(1) = 1   TS(1) = 3.091

N*(2) = 2   TS(2) = 2.028

N*(3) = 3   TS(3) = 1.546

TOOTAL COST PER UNIT TIME = 127.48

OPTIMAL BASIC TIME PERIOD = 7.083

The expression for C_2 in terms of C_{2j}' s is same as it was in Sec. 3.6.3 and DN_j is equal to zero. The total cost per unit time in terms of these variables can be written as:

$$TC = \frac{C_2}{T} + \sum_{j=1}^{M} [a_j + C_{1j} T] \tag{3.12}$$

Solution Procedure:

For given set of N_j' s the optimal value of T is obtained from the following equation:

$$T^* = [C_2/\Sigma C_{1j}]^{1/2} \tag{3.13}$$

If we treat N_j's as continuous variables then we can obtain optimal N_j' s from:

$$\frac{\partial TC}{\partial N_j} = 0$$

$$\Rightarrow \quad N_j^* = [\frac{b_j}{2P_j} (\frac{\beta_j - 1}{\beta_j})]^{1/2} .T$$

Let us denote $[\frac{b_j}{2P_j} (\frac{\beta_j - 1}{\beta_j})]^{1/2}$ by λ_j, then N_j^* can be expressed in terms of N_k^* as:

$$N_j^* = \frac{\lambda_j}{\lambda_k} N_k^*$$

where k is some other equipment. Solution procedure can now be stated in following steps.

Step 1: Obtain λ_j for all the equipments. Select the equipment which has minimum λ_j, let this be r-th equipment.

Step 2: Set $N_r = 1$.

Step 3: Obtain N_j^* for all other equipments using,

$$N_j^* = \frac{\lambda_j}{\lambda_r} \cdot N_r^*$$

Calculate optimal T for these N_j^*'s and then calculate the value of objective function for this set of N_j^*'s and T.

Step 4: $N_r = N_r + 1$, if $N_r > i_{r\ max}$ go to Step 5, otherwise go to Step 3.

Step 5: Select that value of N_r which corresponds to minimum total cost. The corresponding set of N_j^*'s and T will be the optimum solution to the problem.

3.7 Some Parallel Problems:

In this section, we will describe few more problems which use the concepts similar to those, introduced in preceding sections of present chapter.

3.7.1 Multiequipment Production System:

3.7.1.1 Introduction:

In this section, we will consider scheduling for a special type of production system. This system consists of several equipments but production cost per unit produced by the system is a function of effective age of all individual equipments. In this respect all the equipments can not be considered to be operating independently. This is assumed

that the unit production cost for the system can be expressed as summation of polynomial expressions of effective age of all individual equipment. The concepts of effective age and the operation duration, introduced in Chapter I, are still unchanged but we do require to make some modifications in the concept of minor and major repairs to suit the requirements of our present concern. Any equipment can have two kinds of repairs, minor or major. Both of these repairs can bring the equipment to its as good as new condition. When a major repair is carried out all the equipments are repaired simultaneously. The repair duration of any major repair is not negligible compared to the operation durations. In Sec. (3.7.1.3) we will consider the case when these major repair durations can be neglected. Time elapsed between any two consecutive major repairs is known as basic time period for the system. The minor repairs for any equipments are distributed between two consecutive major repairs in such a way that the operation duration between any two consecutive repairs is always constant for that equipment. Duration of these minor repairs are negligible though they also bring the equipments to its as good as new condition.

3.7.1.2 Assumptions:

To further characterize the system we make following assumptions:

(1) Production cost per unit for the system can be expressed as summation of independent polynomial expressions of effective age of all the equipments.

$$f(t) = a + \sum_{j=1}^{M} \sum_{i=0}^{\infty} b_{j,i} \; t_j^i$$

We will further assume that this expression has a simple form given below:

$$f(t) = a + \sum_{j=1}^{M} b_j \, t_j^{n_j}$$

(2) Any kind of repair, minor or major, brings the equipment to its as good as new condition.

(3) Operation durations between any two repairs for an equipment is always constant.

(4) The repair durations for minor repairs are negligible as compared to the operation durations.

(5) The repair duration for major repairs are deterministic and constant.

(6) All the assumptions about the repair costs and downtime costs, as mentioned in Chapter II, still hold good.

(7) No failures due to chance causes are considered.

(8) Effect of deterioration of equipments is to increase the production cost per unit only and it has no effect on the production rate of the equipment.

(1) Production cost per unit for the system can be expressed as summation of independent polynomial expressions of effective age of all the equipments.

$$f(t) = a + \sum_{j=1}^{M} \sum_{i=0}^{\infty} b_{j,i} \; t_j^i$$

We will further assume that this expression has a simple form given below:

$$f(t) = a + \sum_{j=1}^{M} b_j \, t_j^{n_j}$$

(2) Any kind of repair, minor or major, brings the equipment to its as good as new condition.

(3) Operation durations between any two repairs for an equipment is always constant.

(4) The repair durations for minor repairs are negligible as compared to the operation durations.

(5) The repair duration for major repairs are deterministic and constant.

(6) All the assumptions about the repair costs and downtime costs, as mentioned in Chapter II, still hold good.

(7) No failures due to chance causes are considered.

(8) Effect of deterioration of equipments is to increase the production cost per unit only and it has no effect on the production rate of the equipment.

(9) There are ample repair facilities, therefore none of the equipments have to wait for their repair. As a result of this a equipment can have only two states. It can either be operating perfectly or getting repaired.

All other assumptions of Chapter II still hold good for this multiequipment production system. We will be using only those notations in this section, which we have already introduced in previous chapter.

3.7.1.3 Problem Formulation and Solution Procedure:

(a) Problem Formulation:

Production cost per good unit for the system is given by following expression:

$$f(t) = a + \sum_{j=1}^{M} b_j \; t_{ej}^{n_i}$$

where a, n_j's and b_j's are constants for this system and t_{ej} is the effective age of j-th equipment. Figure (3.5) depicts the sequence of events for our system.

In assumption three we have assumed that the operation duration between any two consecutive repairs of any equipment is always constant. It can be proved that even in absense of above assumption if the optimal solution is obtained then that optimal solution will have this property of equal operation duration between any two repairs. This is to say that,

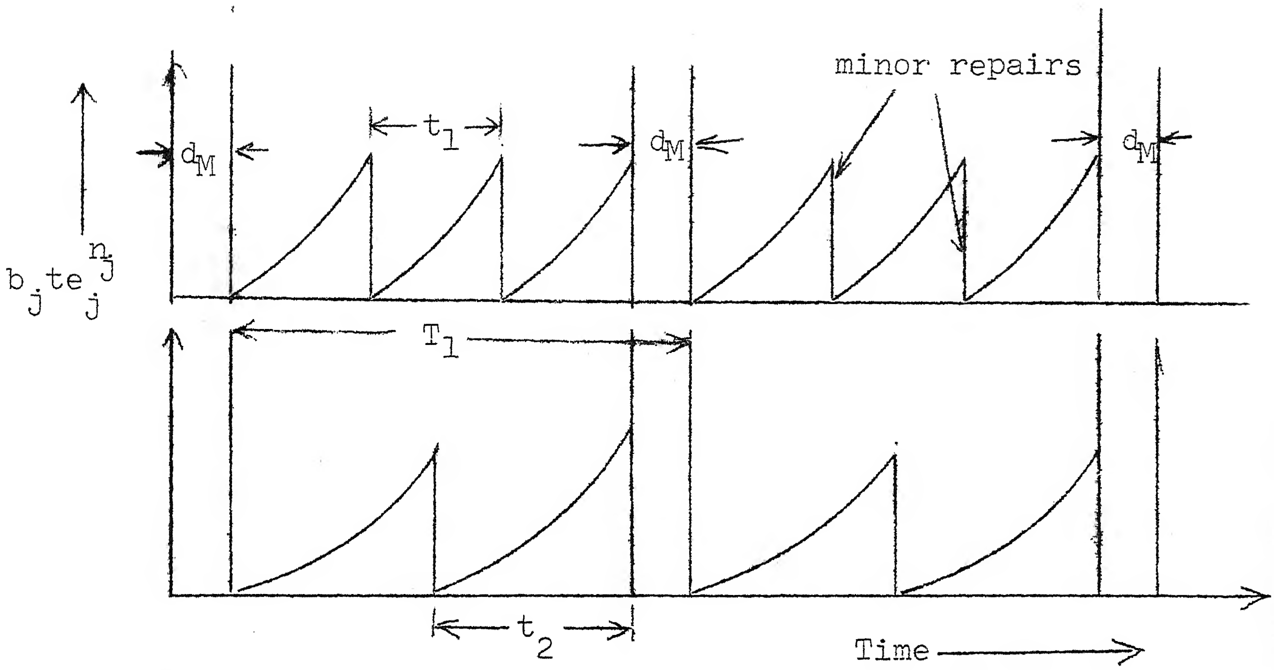

Fig. 3.5: Multiequipment production system (General).

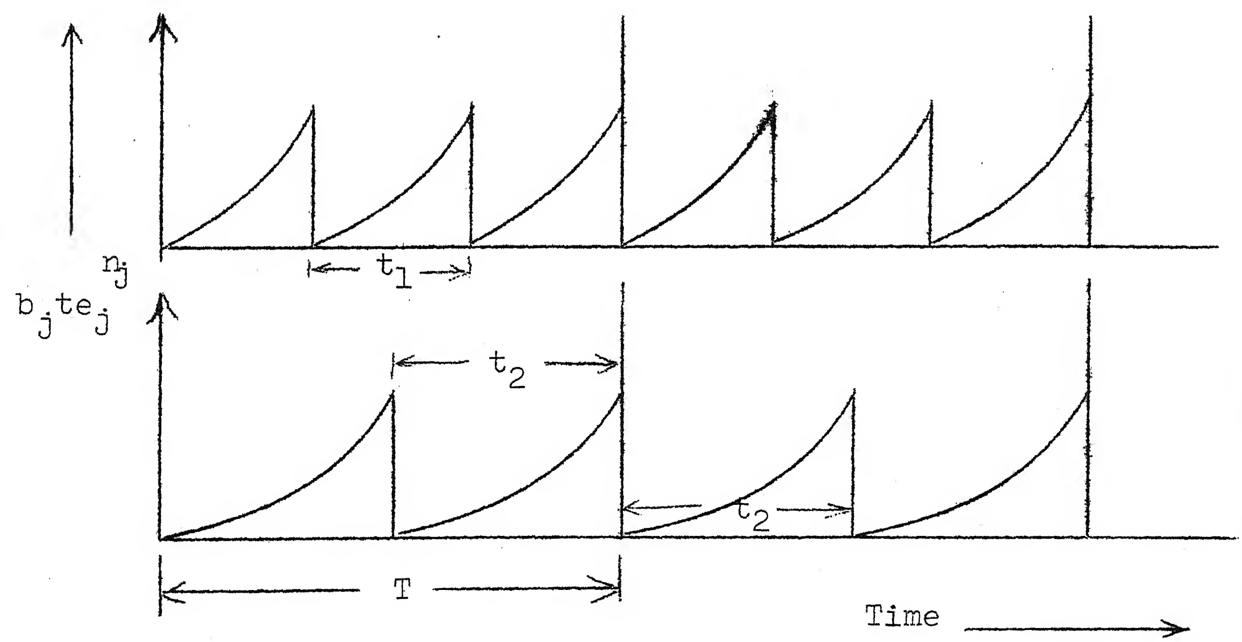

Fig. 3.6 : Multiequipment production system (Special case, $d_M = 0$).

$$t_{j,1} = t_{j,2} = t_{j,3} = \ldots = t_{j,N_j+1} = t_j$$

This t_j is the basic operation duration for equipment j. From Fig. 3.5 a relation between T and t_j can be obtained as:

$$T = (N_j + 1)\, t_j + d_M$$

$$t_j = \frac{T - d_M}{(N_j + 1)}$$

Total cost for the system has three components repair cost, production cost and the downtime cost. Production cost per basic cycle can be written as

$$PC = \sum_{j=1}^{M} [(N_j + 1) \int_0^{t_j} b_j\, t^n\, dt] + a\,(T - d_M)$$

$$= \sum_{j=1}^{M} \frac{b_j}{n+1} \frac{[T - d_M]^{n+1}}{(N_j + 1)^n} + a\,(T - d_M)$$

Repair cost per basic cycle is given by,

$$RC = \sum_{j=1}^{M} P_j\, N_j + A$$

Here A is the major repair cost for the system. The downtime cost per cycle is given by:

$$DTC = R\, d_M$$

Here R is the downtime cost per unit time for the entire system.

Total cost per unit time for the system is given as:

$$TC = \frac{1}{T}[A + d_M (R-a)] + a + \frac{1}{T} \sum_{j=1}^{M} [\frac{b_j}{n+1} \frac{(T-d_M)^{n+1}}{(N_j + 1)^n} + P_j . N_j]$$

(b) Solution Procedure:

The basic time period (T) and the number of minor repairs between two consecutive major repairs (N_j) are our decision variables. Our objective is to minimize the total cost per unit time. To find an initial approximation for the optimal solution we will treat N_j as continuous variables. These values of N_j^*'s can later on be approximated to the nearest integer values for further search.

$$\frac{\partial TC}{\partial N_j} = 0$$

$$\Rightarrow - n \frac{b_j}{n+1} \frac{(T-d_M)^{n+1}}{(N_j^* + 1)^{n+1}} + P_j = 0$$

$$(N_j^* + 1)^{n+1} = (\frac{n}{n+1} . \frac{b_j}{P_j}) (T - d_M)^{n+1}$$

This can be used to write the ratio between $N_j^* + 1$ for two equipments as,

$$\frac{N_j^* + 1}{N_k^* + 1} = (\frac{b_j/P_j}{b_k/P_k})^{1/n+1} \qquad (3.13)$$

Once we are in a position to set a particular value of N_k^* then we can have an approximate value of N_j^* for all other equipment. Similarly for decision variable T, we can write:

$$\frac{\partial TC}{\partial T} = 0$$

$$\Rightarrow -\frac{A + d_M (R-a)}{T^2} + \sum_{j=1}^{M} \left[\frac{b_j}{(n_j+1)(N_j+1)^n} \cdot \frac{(n+1)(T - d_M)^n T - (T - d_M)^{n+1}}{T^2} - \frac{P_j N_j}{T^2} \right] = 0$$

Let,

$$C_1 = \frac{A + d_M (R-a) + \sum_{j=1}^{M} P_j N_j}{\sum_{j=1}^{M} \frac{b_j}{n+1} \cdot \frac{1}{(N_j+1)^n}}$$

then the above equation can be written as,

$$-C_1 + (T^* - d_M)^n [(n + 1) T^* - (T^* - d_M)] = 0$$

$$-C_1 + (T^* - d_M)^n [n T^* - d_M] = 0$$

This cannot be solved explicitely for T^* therefore some iterative methods can be used to obtain the optimal T for given N_j's. The solution procedure can now be stated in following steps.

Step 1: Calculate b_j/P_j for all the equipments. Select the equipment which has minimum b_j/P_j, letk this be equipment r.

Step 2: Set $N_r = 1$

Step 3: (a) Obtain $N_j = \left[\frac{b_j/P_j}{b_r/P_r} \right] [N_r + 1] - 1$ for all the equipments. Find $\underline{N}_j = \lfloor N_j \rfloor$ and $\overline{N}_j = \lceil N_j \rceil$ for all the equipments.

Total cost per unit time can now be written as,

$$TC = \frac{1}{T}\left[A + \sum_{j=1}^{M} P_j N_j\right] + a + \sum_{j=1}^{M} \frac{b_j}{n+1} \frac{T^n}{(N_j+1)^n}$$

To optimize total cost per unit time with respect to the basic time period we set $\partial TC/\partial T = 0$

$$T = \left[\frac{n+1}{n} \frac{A + \sum_{j=1}^{M} P_j N_j}{\sum_{j=1}^{M} \frac{b_j}{(N_j+1)^n}}\right]^{1/n+1} \qquad (3.14)$$

If we proceed exactly in the same way as we did for the general case then following relation between (N_j+1) and (N_k+1) can be obtained.

$$[N_j+1] = \left[\frac{b_j/P_j}{b_k/P_k}\right]^{1/n+1} [N_k + 1]$$

The solution procedure for obtaining the optimal solution for this special case is exactly same as it was for the general case.

3.7.2 Multi-Item Inventory Model:

3.7.2.1 Introduction:

In this section we will consider a multi item inventory problem. Ordering cost functions for these items are similar to the repair cost functions of the production system, which we have considered earlier in this chapter. The items can be ordered either from the whole-sale dealer or from the

producer. There is a fix component of the ordering cost when the items are obtained from the producer directly. This cost component is independent of the number and type of the items which are being ordered. Along with this common component there is a fix component of the ordering cost associated with each item which is being ordered. When, on the other hand, an order is placed to a wholesale dealer then the ordering cost is a simple summation of the ordering cost for each individual items. We have formulated this general problem without giving any solution procedure to solve it because of the complications associated with large number of decision variables.

3.7.2.2 Assumptions and Notations:

(a) Assumptions:

To facilitate the understanding of the system we make following assumptions:

(1) Demand rate for each item is deterministic and constant with respect to time.

(2) Holding cost is linear and is charged on the quantity held per unit time.

(3) No backlogging is allowed.

(4) Lead time is assumed to be constant.

(5) Planning horizon is infinite.

(6) An item r must be ordered, everytime when an order is made to the producer directly. The item r

is that item for which, most frequent ordering to the producer, is made. The time interval between two consecutive such orders is known as basic time period for the system.

(7) Time elapsed between two consecutive major orders of any other equipment is an integer multiple of the basic time period.

(b) Notations:

We will use following symbols for formulating our present problem:

M	Number of items
A	Fixed component of ordering cost when order is made to the producer directly.
A_j	Ordering cost when order is made to the producer.
a_j	Ordering cost when order is made to the wholesale dealer.
h_j	Holding cost per unit per unit time.
C_j	Cost of item
D_j	Demand rate (per unit time)
Q_j	Quantity ordered, when a major order is placed.
q_j	Quantity ordered, when a minor order is placed.
N_j	Number of minor orders placed between two consecutive major orders.
T	Basic time period for the system
T_j	Time interval between two consecutive major orders.
t_j	Time interval between two consecutive minor orders.

In above notations suffix j stands for item j.

3.7.2.3 Problem Formulation:

Ordering and depletion of inventory stock for few of the items is shown in following figure

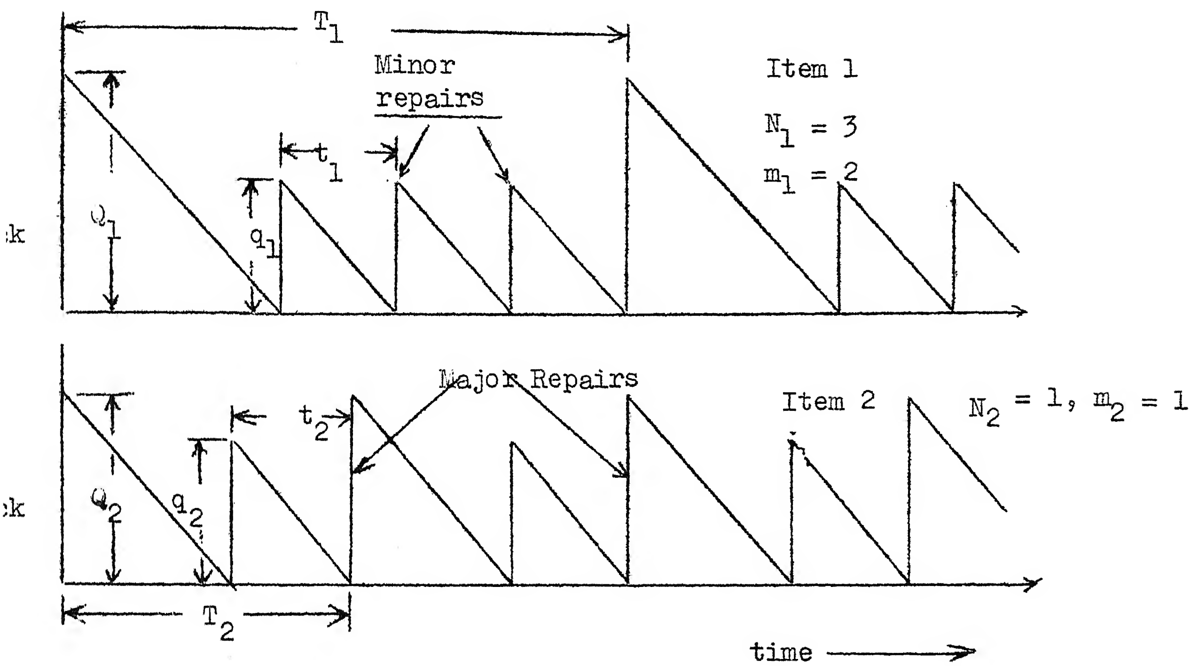

Fig. 3.7: Ordering and depletion of inventory stock.

Total cost per basic time for the system is summation of ordering cost, holding cost and the cost of all the purchased items. Since demand rate for all the items is assumed to be constant and no backlogging is allowed therefore the total cost of all the items which are to be purchased per basic cycle, will remain constant. We will not consider this constant cost for our formulation purposes. Cost of major order per basic cycle can now be written as:

$$MAC = A + \sum_{j=1}^{M} \frac{A_j}{m_j}$$

Similarly cost of minor orders per basic cycles given as:

$$MIC = \sum_{j=1}^{M} \frac{a_j . N_j}{m_j}$$

The holding cost per basic period is given as:

$$HC = \sum_{j=1}^{M} \frac{h_j}{m_j} \{ \frac{Q_j}{2D_j} \cdot Q_j + N_j \cdot \frac{q_j}{D_j} \cdot \frac{q_j}{2} \}$$

$$= \sum_{j=1}^{M} \frac{h_j}{2m_j \; D_j} (Q_j^2 + q_j^2 \; N_j)$$

Now, since all the demand during one basic cycle is to be met without backlogging, we can write:

$$D_j \; T = \frac{Q_j + N_j \; q_j}{m_j}$$

$$N_j = \frac{m_j \; D_j \; T - Q_j}{q_j}$$

The total cost per unit time for the system can now be written as

$$TC = \frac{1}{T} \{A + \sum_{j=1}^{M} [\frac{A_j + a_j \; N_j}{m_j} + \frac{h_j}{2m_j \; D_j} (Q_j^2 + q_j^2 \; N_j)]\} \quad (3.15)$$

where, N_j, m_j, q_j, Q_j and D_j are related by,

$$D_j \; T = \frac{Q_j + N_j \; q_j}{m_j}$$

3.7.2.4 Comment:

For the ordering policy discussed in Sec. 3.8.2.3, obtaining the optimal procurement policy requires computing

the values of four decision variables, namely Q_j, q_j, N_j, m_j for each item and the basic time period for the system. All the four decision variables for any equipment are not independent because they have to satisfy the condition that total demand during a basic time period is equal to the total items procured during that period. No solution procedure for obtaining optimal solution for such systems has been suggested in this section.

3.8 Scope for Future Studies:

Following modifications over the present study can be incorporated in future studies.

(1) That fraction of the total time for which a particular equipment remains in operating state is what determines the average production rate per unit time for that equipment. It is quite possible, that under certain circumstances the production rate may not be allowed to fall below a particular predefined level. The general repair scheduling problem for a production system, therefore, should aim at minimizing the total cost per unit time, under the constraints that the availability level for any equipment does not violate the availability constraints. The total cost expression for such systems will include only the production cost and the repair cost.

The general form of this problem can be expressed as:

Objective : To minimize the total cost per unit time for the system.

Subject to: Availability constraints.

The objective function can be written as:

Objective function = TC = Production cost per unit time + repair cost per unit time.

The availability constraints are usually of the form:

availability for equipment, $\geq x_j \;\forall\; j$

x_j is tolerable availability level for equipment j.

(2) In most of the practical situations the number of repair facilities are also one of the decision variables. This is usually decided on the basis of the fraction of the total time for which a equipment waits in the queue before getting a chance for repair, the cost associated with this waiting time and the machine utilization factors etc. Though for our present study we have assumed ample number of repair facilities, for future studies it may be treated as a decision variable.

(3) In our solution procedure we have assumed that the operation duration between any two consecutive repairs is constant. This assumption was intended to simplify the formulation for our problem. Since this was an additional constraint on our system, it led to a suboptimum solution. For the cases when $n > 1$ and $\beta < \infty$ it can be proved that the

optimum repair duration between i-th and (i+1)-th minor repair is always less than the optimal operation duration between (i-1)-st and i-th minor repair. Under such circumstances instead of treating t_j as a decision variable for the operation durations between various two consecutive minor repairs, we may try to optimize the objective function with respect to one or more decision variables, known as parameters of operation-duration-series. Each term of the series represented by these parameters gives the operation durations between two consecutive repairs of that equipment. For example, one may consider the series:

$$t_j\ ,\ \ (\tfrac{3}{4})\ t_j,\ (\tfrac{3}{4})^2\ t_j,\ (\tfrac{3}{4})^3\ t_j,\ \ldots$$

with only one operation duration parameter, namely t_j, to represent . the operation durations, between two consecutive repairs. t_j is the operation duration between a major repair and first minor repair $(\frac{3}{4})\ t_j$ is the operation duration between first and second minor repairs and so on. An another series with two parameters, to represent the operation durations could be:

$$t_j,\ at_j,\ a^2t_j,\ a^3t_j,\ \ldots$$

Many similar series can be considered of depending upon the characteristics of the equipment j.

Above three changes can be incorporated in future studies, concerning scheduling of repairs for production systems.

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$$t_j, \quad (\tfrac{3}{4})\, t_j, \quad (\tfrac{3}{4})^2\, t_j, \quad (\tfrac{3}{4})^3\, t_j, \ \ldots$$

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$$t_j, \ at_j, \ a^2t_j, \ a^3t_j, \ \ldots$$

Many similar series can be considered of depending upon the characteristics of the equipment j.

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CHAPTER IV

REPAIR POLICY FOR SERVICE PRODUCING SYSTEM

4.1 Introduction:

For goods producing systems the repair policy is usually decided on the basis of total cost per unit time alone, whereas for a service producing systems the repair policy is decided on the basis of both, the minimum reliability level of the system (or individual equipment, in some special cases) at any instant, when it is in operating state, and the cost per unit time. In this chapter we will present a mathematical model and solution procedure for obtaining an optimal repair policy for a service producing systems. The objective of the problem is to minimize the total cost per unit time under given set of reliability constraints for the system. To simplify the situation we will not consider any random breakdowns of the components or equipments. The concept of reliability should, therefore, be understood as the probability that the equipment has not deteriorated beyond a particular level. We do not, always, have to go for an urgent repair when the operating characteristics of the system (or of individual equipment) deteriorates beyond the above mentioned minimum level because the equipment can still be operating, though at a level which is worse than the recommended one. A close

resemblance can be observed between the system mentioned above and the one, which we analyzed in Chapters II and III.

In general the problem mentioned above is very difficult to handle because for most of the cases, the reliability constraints on the system can not be easily translated in terms of reliability constraints of the individual equipments. The degree of difficulty in above translation increases exponentially with any increase in the degree of complications of the system. To simplify the situation, we will consider only those cases for which the reliability constraints of the system can be translated in terms of the reliability constraints of the individual equipments. We will further assume that the reliability constraints of individual equipments, as obtained after above translation, are such that at any instant of time the Lowest Acceptable Reliability Level (LARL) for a equipment does not depend upon the reliability level of other equipments of the system, at that time.

For any equipment, while in operation, reliability keeps decreasing with time. This decrease in the reliability of the equipment can be attributed to deterioration of the operating characteristics of that service producing system. A repair operation will improve the reliability of the system. Here again we will consider two types of repairs, major and minor. A major repair will bring the reliability of the equipment to one, whereas a minor repair will bring the

reliability level of the equipment to level which is less than one. The effect of a minor repair is to reduce the effective age of the equipment by a constant factor, known as improvement factor.) The objective of the problem, is to find a repair policy with minimum cost per unit time under the constraints that the reliability level of any equipment at any instant of time does not fall below the lowest acceptable reliability level of that equipment.

The total cost function for this problem has only two component, the repair cost and the downtime cost. Both of these cost components keep decreasing with any decrease in the number of repairs. The decision variables include the number of major repairs per unit time (or the time interval between two consecutive major repairs) and the number of minor repairs between two consecutive major repairs. The number and spacing of minor repairs between two consecutive major repairs will always remain the same throughout the life of the equipment because of the deterministic nature of the problem.

We will first consider the case when all the equipments can be treated individually for calculating the total cost per unit time. The total cost for the system can be expressed as the sum of total cost for all the individual equipments. This case will be considered in Sec. 4.3. Similar to Chapter III, when the set-up cost is not zero, the

corresponding repair policy is termed as mixed repair policy and when all the equipments are always forced to have their major repairs simultaneously then the corresponding repair policy is termed as joint repair policy. Mixed repair policy and joint repair policy along with their solution procedures are discussed in Sec. 3.4 and 3.5 respectively. To illustrate the solution procedures, numerical examples, for all above cases, have also been considered in the respective sections.

4.2 Assumptions and Notations:

4.2.1 Assumptions:

In addition to the assumptions concerning number of repair facilities, repair durations, repair costs and down-time costs (Assumption 3, 4, 5, 6) in Chapter II, we make following assumptions.

(1) There is no interaction between the hazard functions of any two equipments, that is, the reliability level of one equipment does not have any effect on the reliability level of the others.

(2) All the equipments, when new, have the reliability level equal to one.

(3) The reliability of any equipment is completely deterministic and is a function of the effective age of the equipment only.

(4) Any major repair brings the reliability level of the equipment to one, that is, the equipment behaves as good as new . Any minor repair reduces the effective age of the equipment by a constant factor, known as improvement factor for the equipment. For a given equipment the improvement factor remains constant for all the minor repairs.

(5) The parameters a_j, b_j and c_j of the hazard function (see section 4.2.2) of the equipment j remain constant throughout the life of the equipment.

(6) Reliability constraints of the system as a whole can be translated into the reliability constraints for the individual equipments. The reliability constraints on the individual equipments, as obtained from the system constraints, are such that at any instant of time the reliability for a equipment is not allowed to fall below the LARL for that equipment. The LARL for any equipment is independent of LARL for other equipments.

(7) When the reliability level for any equipment touches the lowest acceptable reliability level (LARL) then either a minor repair or a major repair will take place. This assumption also emphasises that the equipment can have only two states, either it will be operating property or it will be getting repaired.

4.2.2 Notations:

In this chapter we will be using following notations in addition to those which were introduced in Chapter II.

For equipment j:

$h_j(t)$ = hazard function

= $a_j + b_j t^{c_j}$

where a_j, b_j and c_j are characteristics of the equipment j. All these three are assumed to remain constant throughout the life of the equipment (Assumption (4)].

$Re_j(t)$ = Reliability (or reliability level) at time t.

$t_j(i,0)$ = Time, from the end of most recent major repair till the beginning of i-th minor repair.

$t_j(i,1)$ = Time from the end of most recent major repair till the end of i_m minor repair.

G_j = Time taken by new equipment till its reliability drops down to the lowest acceptable reliability level (LARL), assuming that no repairs takes place during this period.

$g_j(i,1)$ = Effective age at the end of the i-th minor repair.

$g_j(i,0)$ = Effective age at the beginning of the i-th minor repair.

α_j = Lowest acceptable reliability level (LARL).

4.3 Individual Repair Policy:

In this section we will consider a machine shop which consists of several independently operating servicing equipments.

It is assumed that the set up cost for their repair is very small. As a result of this, there can not be any saving in the set-up cost by scheduling two or more components simultaneously for their major repairs. Besides, its direct application for the situation mentioned above, this analysis would perhaps be useful for the study of mixed and joint repair cases.

4.3.1 Problem Statement:

Assumption 1 enables us to write the hazard function for each equipment independently. Let the hazard function for equipment j be written as:

$$h_j(t) = a_j + b_j t^{c_j}$$

Let us now define three new variables,

$$\mu_j = a_j$$
$$\eta_j = c_j + 1$$
$$\theta_j = b_j/[c_j + 1)$$

In terms of the effective age of the equipment j at time t $[g_j(t)]$, the reliability of the equipment j can be given by:

$$Re(t) = \exp[-\mu_j g_j(t) - \theta_j . g_j(t)^{\eta_j}]$$

The effective age of the equipment j just before and after i-th minor repair after the most recent major repair can be written as:

$$Re_j(i,0) \;=\; \exp\,[-\mu_j\, g_j\,(i,0) - \theta_j\,[g_j(i,0)]^{\eta_j}]$$

$$Re_j(i,1) \;=\; \exp\,[-\mu_j\, g_j\,(i,0) - \theta_j\,[g_j(i,0)]^{\eta_j}]$$

where $g_j(i,0)$ and $g_j(i,1)$ have their usual meaning.

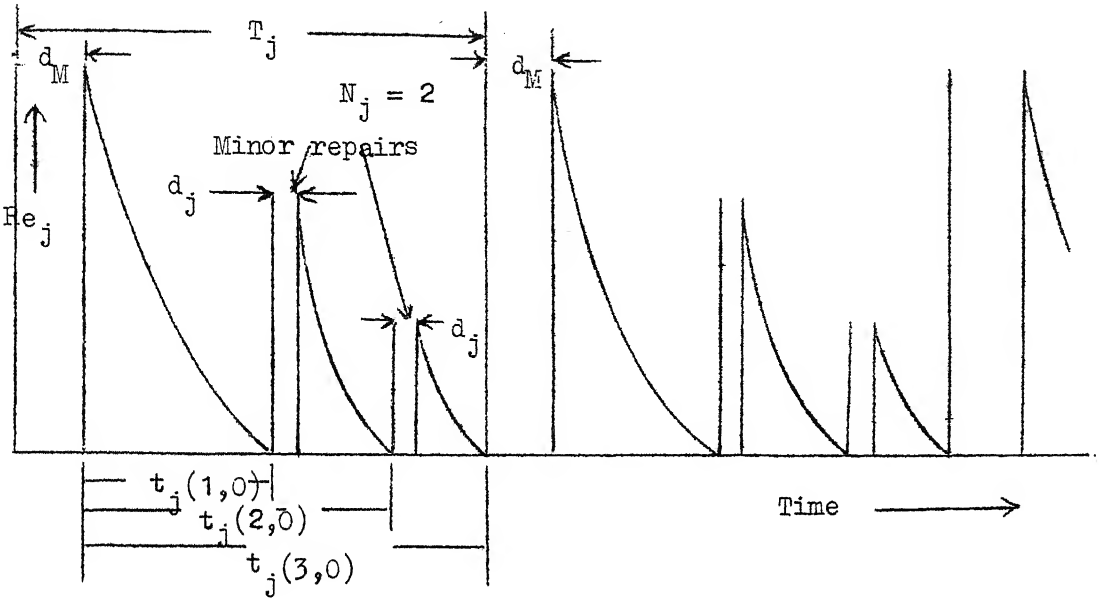


Fig. 4.1: Sequence of events for equipment j.

With the help of Fig. 4.1 and the concept of effective age, introduced in Chapter I, we can write the expression for effective age of equipment, at any instant between (i-1)-st and i-th minor repair (after the most recent major repair) as:

$$g_j(i,\,t) \;=\; \frac{t_j(1,\,0)}{\beta_j} + \frac{t_j(2,0) - t_j(1,0) - d_j}{\beta_j}$$

$$+ \;\ldots\ldots\; + [t - t_j\,(i-1,0) - d_j)$$

Effective age of the equipment j at the beginning of i-th minor repair and at the end of i-th minor repair

respectively can be written as:

$$g_j(i, 0) = \begin{cases} \dfrac{t_j(i-1,0)}{\beta_j} - \dfrac{(i-2)\, d_j}{\beta_j} - d_j + [t_j(i,0) - t_i(i-1,0) & \forall\, i \geq 2 \\ t_j\,(i,0) & \text{for } i = 1 \end{cases}$$

$$g_j(i,1) = \frac{t_j(i,0)}{\beta_j} - (i-1)\,\frac{d_j}{\beta_j}$$

When the equipment is in working state, but its reliability level has dropped down to the lowest acceptable reliability level (α_j) then following should hold good:

$$\alpha_j = \exp\,(-\mu_j\, G_j - \theta_j (G_j)^{\eta_j})$$

$$\text{Log}\ \alpha_j = -\mu_j\, G_j - \theta_j\, G_j^{\eta_j} \qquad (4.2)$$

Above equation cannot be solved to render an expression for G_j explicitely in terms of other variables. Any iterative method can be employed to solve G_j numerically. For the sake of simplicity in the mathematical formulation we have assumed (Assumption 7) that the equipment can have only two states. Either it will be functioning properly or it will be getting repaired. It can be proved logically that even if we delete above assumption our optimum solution can never allow the equipment to be in any state other than the two, mentioned above. To prove it we will assume that for our optimum solution (say solution I) the number of minor repairs between two consecutive major repairs be N_j and the time period be T_j for equipment j. Let us further

assume that even when the equipment is not getting repaired, it remains in non-operating state for x time units per cycle. Let the cost for this optimum solution per cycle be denoted by K_1. Now let us consider another solution, feasible one (say solution II), in which the number of minor repairs between two major repairs be still N_j; but the time period be reduced to $(T_j - x)$. The equipment is not allowed to remain in nonoperating state when it is not getting repaired. Obviously the cost per unit cycle for this case can be written as:

$$K_2 = K_1 - x \cdot R_j$$

Cost per unit time for solution I and solution II can now be written as:

$$TC_1 = \frac{K_1}{T} \quad \text{and} \quad TC_2 = \frac{K_2}{T-x} = \frac{K_1 - xR_j}{T-x}$$

For any solution to be optimal following condition must hold good (otherwise it will always be preferable, not to have any repair at all)

$$T_j R_j > K_1$$

$$x . T_j R_j > x K_1$$

$$T_j K_1 - x T_j R_j < T_j K_1 - x K_1$$

$$T_j (K_j - xR_j) < K_1 (T_j - x)$$

$$\frac{K_1 - x R_j}{T_j - x} < \frac{K_1}{T_j}$$

$$TC_2 < TC_1$$

This inequality clearly states that the solution II is better than solution I. Thus we can safely claim that the equipment can not be in nonoperating condition, when it is not getting repaired, in the optimum solution.

Using the fact that all the minor repairs are of d_j duration, we can schedule the beginning of minor repairs as:

$$
\begin{aligned}
t_j(1, 0) &= G_j \\
t_j(2, 0) &= G_j + d_j + t_j(1,0).[1- \frac{1}{\beta_j}] \\
t_j(3,0) &= G_j + d_j + t_j(2,0).[1- \frac{1}{\beta_j}] \\
&\quad + \frac{d_j}{\beta_j} \\
&\vdots \\
t_j(i,0) &= G_j + d_j + t_j(i-1,0) . \\
&\quad [1 - \frac{1}{\beta_j}] + \frac{(i-2)\ d_j}{\beta_j}
\end{aligned}
\qquad (4.3)
$$

where $t_j(i,0)$ indicates the scheduled beginning of i-th minor repair. The next major repair will take place at $t_j\ (N_j + 1,\ 0)$ time units after the previous major repair, thus the cycle time for equipment j can be written as:

$$T_j = t_j\ (N_j + 1,\ 0) + d_M \qquad (4.4)$$

Downtime for equipment j between two consecutive major repairs can be written as:

$$DT_j = \frac{N_j d_j + d_M}{T_j} + R_j$$

Total repair cost for equipment j can be written as:

$$RC_j = \frac{1}{T_j} [N_j d_j + AS_j]$$

For equipment j the total cost per unit time can now be written as:

$$TC_j = DTC_j + RC_j$$

$$= \frac{1}{T_j} [(N_j d_j + d_M) R_j + N_j P_j + AS_j] \qquad (4.5)$$

Total cost for the system can be expressed as simple sum of total cost for each individual equipment.

$$TC = \sum_{j=1}^{M} TC_j$$

4.3.2 Solution Procedure:

From the expression for total cost per unit time it can be easily seen that the optimal repair schedule for the system as a whole, can be obtained by obtaining the optimum repair schedule for each equipment individually. Using Eqn. (4.4), the expression for total cost per unit time for equipment j, can be written as:

$$TC_j = \frac{1}{t_j [N_j+1, 0] + d_M} [N_j[d_j R_j + P_j] + [d_M R_j + AS_j]]$$

For obtaining the expression for T_j explicitely in terms of N_j s we define a new array x_i as:

$$x_1 = t_j\ (1, 0).$$

$$x_2 = t_j\ (2, 0) - t_j\ (1, 0) - d_j$$

$$\vdots$$

$$x_{N_j} = t_j\ (N_j, 0) - t_j\ (N_j - 1, 0) - d_j$$

The expression for G_j can now be written in terms of x_i' s and β_j as,

$$G_j = x_1$$

$$G_j = \frac{x_1}{\beta_j} + x_2$$

$$\vdots$$

$$G_j = \frac{x_1 + x_2 + x_3 + \ldots + x_{N_j}}{\beta_j} + x_{N_j+1}$$

From the above expression x_j can be obtained as:

$$x_j = G_j\ (1 - \frac{1}{\beta_j})^{i-1}$$

Now the time period can be expressed in terms of x_i's as

$$T_j = x_1 + x_2 + \ldots + x_{N_j+1} + N_j d_j + d_M$$

This finally yields,

$$T_j = G_j\ [1 - (1 - \frac{1}{\beta_j})^{N_j+1}]\ \beta_j + N_j\ d_j + d_M$$

The total cost expression now becomes:

$$\frac{A_j + [P_j + R_j\, d_j]\, N_j + R_j\, d_M}{G_j\, \beta_j\, [1-(1-\frac{1}{\beta_j})^{N_j+1}] + N_j d_j + d_M}$$

Now, the total cost per unit time for equipment j is function of only one decision variable, N_j. N_j can assume any positive integer value. From equation 4.3, we can derive following inequalities:

$$t_j\,(N_j+1,\ 0) \geq t_j\,(N_j,\ 0) \geq t_j\,(N_j - 1,\ 0)$$

and $$t_j\,(N_j+1,\ 0) - t_j\,(N_j,\ 0) \leq t_j\,(N_j,\ 0) - t_j\,(N_j-1,\ 0)$$

Using these inequalities it can be proved easily that the total cost per unit time, for equipment j, is unimodular in N_j. For obtaining the optimal value of N_j, we can make a one dimensional search along the integer values of N_j. The optimal N_j will satisfy following inequalities,

$$TC_j\,(N_j^*) \leq TC_j\,(N_j^* - 1)$$

$$TC_j\,(N_j^*) \leq TC_j\,(N_j^* + 1)$$

The solution procedure now can be stated in following steps.

1. Set $N_j = 0$
2. Calculate $TC_j\,(N_j)$
3. If $N_j = 0$ then $TC_j^* = TC_j\,(N_j)$

   otherwise if $TC_j\,(N_j)$ is less than TC_j^* then

   set $N_j = N_j + 1$ and go to Step 2,

   otherwise go to Step 4.

4. The optimal value of the decision variables are given by,

$$N_j^* = N_j + 1, \quad \text{and}$$

$$T_j^* = t_j\,(N_j^* + 1,\ 0) + d_M$$

To illustrate the solution procedure we will take up a numerical example in Sec. 4.3.3.

4.3.3 Numerical Example:

Let us consider a service producing system with three equipments. Related data is given on Table 4.1. Time, when a new equipment hits its LARL is obtained from Eqn. (4.2).

These values of G_j s are as follows:

$$G_1 = 1.445$$

$$G_2 = 1.507$$

$$G_3 = 1.038$$

We select i_{max} for all the equipments as 5 and calculate the values of t_j (i, 0) from equation 4.3. Table 4.2 gives the values of t_j (i, 0) for j = 1, ..., M and i = 1, ..., 25. To find the optimal solution we will vary N_j and calculate total cost for each equipment independently and then we will select that value of N_j, for each equipment, which corresponds to the minimum cost for that equipment. Table 4.3 gives optimal T_j and the corresponding TC_j, for each value of N_j, for all the equipments. This optimal solution is given in Table 4.4.

TABLE 4.1: INPUT DATA FOR INDIVIDUAL REPAIR POLICY EXAMPLE.

NUMBER OF EQUIPMENTS = 3

DURATION OF MAJOR REPAIR = 1.5

EQUIPMENT	C	AS	BF	AM(I)	PH(I)	AL(I)	ET	P(I)	R(I)
1	100.00	45.0	5.	0.020	0.080	0.700	3.	0.80	50.00
2	100.00	60.0	1.	0.020	0.030	0.800	3.	0.20	100.00
3	110.00	35.0	3.	0.120	0.120	0.300	3.	0.40	140.00

TABLE 4.2: TIME, AFTER MOST RECENT MAJOR REPAIR, TILL THE BEGINNING OF T-TH MINOR REPAIR OF I-TH EQUIPMENT.

I \ T	1	2	3	4	5	6	7	8	9	10
1	1.445	2.900	4.125	5.164	5.956	6.830	7.500	8.411	8.554	9.147
2	1.567	2.237	3.334	4.720	5.307	5.954	6.422	6.823	7.174	7.487
3	1.038	1.990	2.391	2.779	3.103	3.340	3.531	3.692	3.832	3.959

4.3: OPTIMAL T(J) AND TC(J) FOR EACH GIVEN N(J).

T(1)	TC(1)	T(2)	TC(2)	T(3)	TC(3)
2.045	34.24	2.107	56.96	1.638	100.14
3.500	32.85	3.437	46.56	2.429	77.38
4.725	33.86	4.484	44.60	2.991	70.89
5.764	35.56	5.320	45.11	3.398	69.45
6.656	37.56	5.997	46.69	3.703	70.21
7.430	39.71	6.554	48.82	3.940	72.09
8.108	41.93	7.022	51.27	4.131	74.56
8.711	44.20	7.423	53.88	4.292	77.36
9.254	46.47	7.774	56.60	4.432	80.32
9.747	48.73	8.087	59.35	4.559	83.35
10.203	50.97	8.372	62.11	4.677	86.38
10.627	53.17	8.636	64.85	4.789	89.37
11.026	55.32	8.884	67.54	4.897	92.30
11.405	57.43	9.119	70.18	5.002	95.16
11.769	59.48	9.346	72.76	5.106	97.93
12.120	61.47	9.566	75.26	5.208	100.61

4.4: THE OPTIMAL SOLUTION
(INDIVIDUAL REPAIR POLICY).

THE OPTIMAL SOLUTION

:1) = 1 T*(2) = 3.500 TC*(1) = 32.85

:2) = 2 T*(2) = 4.484 TC*(2) = 44.60

:3) = 3 T*(3) = 3.398 TC*(3) = 69.45

'AL OPTIMAL COST = 146.91

4.4 Mixed Repair Policy:

4.4.1 Introduction:

In this section, we will develop a mathematical model and solution procedure for a group of equipments which, when repaired together can share a common component of major repair cost. It is because of this common component of the major repair cost, that scheduling of two or more major repairs together, may lead to a more economic repair schedule than individual repair schedules. As a special case we set this common component of major repair cost equal to zero, we get the same problem as we have discussed in Sec. 4.2 and 4.3.

We define m_j as number of major repairs of equipment r for each major repair of equipment j (this is similar to what we used in Chapter III). The equipment r is that equipment which has maximum number of major repairs in a given time interval. Obviously the time between two consecutive major repairs of equipment j will be m_jT, where T is the time between two successive major repairs of equipment r. Our objective is to minimize the total cost per unit time under the constraint that the reliability level of any of the equipment does not fall below the lowest acceptable reliability level for that equipment. Our decision variables are T, the basic time period, m_j's, the number of major repairs

of equipment r for each major repair of equipment j, and N_j' s, the number of minor repairs of equipment j between two consecutive major repairs of this equipment. Since nature of the problem is deterministic, all the equipments will repeat the same repair schedule after a fixed cycle time.

For the general case, mentioned above, we will modify the assumptions in Sec. 4.4.2. In Sec. 4.4.3, we will consider the problem formulation and solution procedure. A numerical example for this policy will be considered in Sec. 4.4.5.

4.4.2 Assumptions:

For the case of mixed repair scheduling the assumption seven of Sec. 4.2 can be restated as:

(7') When the reliability level of a equipment touches the lowest acceptable reliability level then either it will have a repair (minor or major) or it will wait till the scheduled beginning of next major repair. How far away is the scheduled beginning of next major repair, is what decides the appropriate course of action, that is, whether the equipment should wait or get repaired.

In addition to the assumptions of Sec. 4.2 we make following assumptions concerning synchronization of major repairs.

(8a) The time between two consecutive major repairs of any equipment is an integer multiple of the basic time period.

(8b) Basic time period is the time between two consecutive major repairs of equipment r, where equipment r, is the equipment which has most frequent major repairs.

(8c) Any major repair can take place only when the equipment r is having a major repair.

4.4.3 Problem Formulation and Solution Procedure.

4.4.3.1 Problem Formulation:

Our system consists of several equipments operating independently. The reliability constraints for all these equipments are also known independently. Fig. 4.2 shows the sequence of events for these equipments.

It is quite possible that in many cases $(t_j(N_j+1, 0)+d_M)$ may not be an integer multiple of $(t_r(N_r+1, 0) + d_M)$. For such cases either the equipment j and/or r will have remain in non-operating state for sometime or the equipment j and/or equipment will have to under go the common repair operation even before touching the respective lowest acceptable reliability level. The term δ, as shown in the figure can therefore be written as,

$$= \max\,(0,\ m_j T - d_M - t_j\ (N_j + 1,\ 0))$$

When for a equipment the reliability level has dropped down to the LARL then three possible courses of action can be

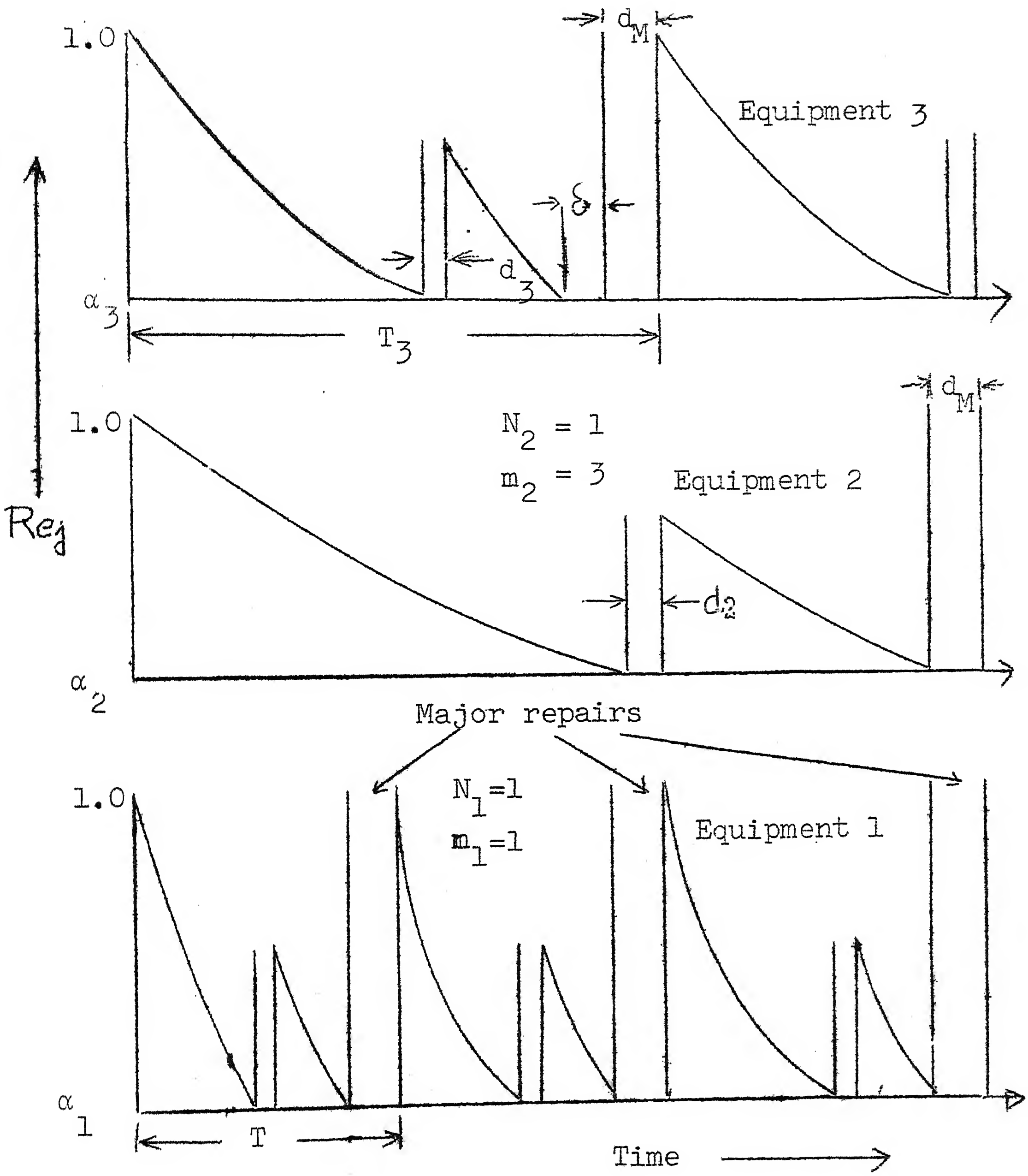


Fig. 4.2: Mixed repair policy for a service producing system.

selected. We may start a minor repair or a major repair or we may wait for some time before starting a common major repair operation. Any of these possible courses of action can be taken depending upon, how far, the beginning of next scheduled major repair is, from the time, when LARL is reached. The following rule can be followed to simplify the mathematical formulation.

Rule 1 (a) If the beginning of next major repair is scheduled at more than d_j time units then a minor repair should be taken up.

(b) If the beginning of next major repair is scheduled right at the instant when reliability level touches the LARL then a major repair should start.

(c) If the beginning of next major repair is scheduled at less than d_j time units then the equipment should wait till the scheduled beginning of next major repair.

A little thinking will reveal that the above rule is imposing an additional constraint on the system and therefore we may finally end up with a solution which is slightly away from the optimal one. Nevertheless if we consider the simplification in formulation, the importance of rule one becomes quite apparent. In Sec. 4.6 we will discuss the situations where rule one is not applied to our system.

To maintain the repeatability of the cycles, it is necessary to follow an another rule, rule two. Rule two can be stated as following:

Rule 2: When the time for next scheduled major repair has arrived, then we will start with next major repair even if the system has reliability level higher than α_j, the lowest acceptable reliability level.

Similar to Sec. 4.3.1, the time when the reliability level of a equipment (just after a major repair) touches the LARL, can be calculated using Eqn. (4.2). To solve this equation for obtaining the numerical value of G_j, we can use any iterative technique. Using the concept of effective age and the definition of G_j, we can find the timings for the beginning of minor repairs from Eqn. (4.3) of Sec. (4.3.1).

Now using the rules one and two we can state that the equipment j will have N_j minor repairs between two consecutive major repairs if and only if the following conditions are satisfied.

$$t_j\ (N_j,\ 0) \leq m_j T - d_M - d_j$$

and $$t_j\ (N_j + 1,\ 0) > m_j T - d_M - d_j \qquad (4.6)$$

Here the terms have their usual meanings. Downtime for the equipment j between two consecutive major repairs can be written as:

$$DT_j = n_j d_j + d_M + \max [0, m_j T - d_M - t_j (N_j + 1, 0)]$$

fractional downtime for equipment j $= \dfrac{DT_j}{m_j T}$

Downtime cost per unit time for all the equipments can now be written as:

$$DTC = \sum_{j=1}^{M} R_j \frac{DT_j}{m_j T}$$

The total repair cost per unit time, for major and minor repairs of all the equipments can be written as:

$$RC = \frac{1}{T} [\sum_{j=1}^{M} \frac{P_j N_j}{M_j}] + [A + \sum_{j=1}^{M} \frac{AS_j}{M_j}] \frac{1}{T}$$

Total cost per unit time for the system can be written as:

$$TC = DTC + RC$$

$$= \frac{1}{T} [A + \sum_{j=1}^{M} \{ \frac{P_j N_j}{M_j} + \frac{AS_j}{M_j} + R_j \frac{DT_j}{M_j} \}] \qquad (4.7)$$

4.3.3.2 Solution Procedure:

It is very difficult to handle the objective function mathematically, because of the occurrance of the term $\max (0, m_j T - d_M - t_j (N_j + 1, 0))$, however, this can be seen that the optimal solution will always correspond to the situation where atleast for one of the equipments $\max (0, m_j T - d_M - t_j (N_j + 1, 0))$ is zero. In case, when for all the equipments this terms is nonzero then we can reduce the downtime cost for all the equipments by reducing the basic time period

till for atleast one of the equipments max $(0,\ m_jT-d_M-t_j(N_j+1,\ 0))$ becomes zero. We also note that by doing this we do not incur any additional repair cost per basic cycle, but the contribution of repair cost in the total cost per unit time increases because of the decrease in the basic cycle time. It has been proved in Appendix A that the new solution, obtained by reducing the basic time period as mentioned above, will yield a better solution than the previous one. The proof of above statement is entirely based upon the fact that if we are at all going for any repair policy then that repair policy must certainly be more economical than the policy of allowing all the equipments to remain in nonoperating state throughout their life, after they once reach their lowest acceptable reliability levels.

With the help of above statement, we do, no longer, have to make a thorough search for obtaining optimal value of the basic time period. The basic time period can now be written as:

$$T \ \varepsilon \ \left\{ d_M + t_r(i_r,\ 0)\ ,\ \frac{D_M + t_j(i_j,0)}{m_j} \right.$$

$$i_r = 1,2,\ldots,\ i_{r\ max},\quad i_j = 1,2,\ldots,\ i_{j\ max},$$

$$m_j = 1,2,\ldots,\ m_{j\ max} \tag{4.8}$$

Where equipment r is the one which is having most frequent major repairs. $i_{r\ max}$ and $i_{j\ max}$ are the empirical upper limits on the number of minor repairs between two major repairs for equipment j. We know that as N_j goes up the operation duration between two consecutive minor repairs $[t_j(N_j, 0) - t_j(N_j+1, 0) - d_j]$ goes down. If we do not want the operation duration to go below a particular level than that will automatically fix the value of $i_{r\ max}$ and $i_{j\ max}$. This restriction on minimum allowable operation duration, in some sense, gives an indication about the values of $m_{j\ max}$ for other equipments also.

For finding the optimal solution we select a combination of m_j's, such that atleast one of them is 1. We then select an equipment for which max $(0, m_1T - t_1\ (N_1+1, 0) - d_M)$ is to be kept at zero. Now we choose N_j for this equipment. This selection of N_1 automatically fixes the value of the basic time period T. Since we have already selected a set of m_j s therefore the values of N_j for all other equipments can now be obtained from the following inequalities:

$$\begin{aligned} m_jT - d_j - d_M &\geq t_j\ (N_j, 0) \\ m_jT - d_j - d_M &< t_j\ (N_j+1, 0) \end{aligned} \qquad (4.9)$$

Knowing m_j's, N_j's and T we can calculate the total cost corresponding to these values of decision variables. We now vary the value of N_1 for the equipment 1, which we have already

selected and calculated corresponding m_j s and N_j s for all other equipments and the basic operation time for the system. Using these, equation 4.7 can be used to obtain the total cost. We now select, one by one, all other equipments for which $\max(0, m_jT - t_j(N_j+1, 0) - d_M)$ should be zero and repeat the calculation of total cost for all values of N_j as we did for the case of equipment one.

We repeat above procedure for all other possible combinations of m_j's and finally select the one which gives the best result. The corresponding m_j's, T and N_j's will be the optimal values of the decision variables. This solution procedure can be stated in following steps:

<u>Step 1</u>: Calculate G_j's and $t_j(i, 0)$'s for all the equipments $(0 \leq i \leq i_{j\,max})$

<u>Step 2</u>: Select a set of m_j such that $\exists_j : m_j = 1$.

<u>Step 3</u>: Set $k = 1$, $N_k = 0$

<u>Step 4</u>: Set T as $T = t_k(N_k+1, 0) + d_M$ and N_j's from the inequalities 4.9. Calculate total cost from (4.7).

<u>Step 5</u>: $N_k = N_k+1$, If N_k is greater than $i_{k\,max}$, go to Step 4, otherwise $k = k+1$. If k exceeds M (the number of equipments) then go to Step 6, otherwise set $N_k = 0$ and go to Step 4.

<u>Step 6</u>: Select another set of m_j's such that $m_j = 1$ for atleast one of the equipments. If no such combination is possible, go to Step 7, otherwise go to Step 3.

Step 7: Select that set of decision variables which gives the minimum total cost for the system.

To illustrate the solution procedure for a mixed repair case with reliability constraints, we take up following example.

4.4.4 Numerical Example:

Let us consider a system with three service producing equipments. The reliability constraints for all the three equipments has a form such that the reliability for none of the equipments is allowed to fall below a particular predefined level, known as LARL for that equipment. The equipment characteristics and the system parameters are given in Table 4.5. Since the equipment characteristics for this example are exactly same as they were for individual repair policy problem, the values of G_j's and t_j (i, 0)'s remain the same, as they are given in that example. For all possible sets of m_j's, and for each such set of m_j's, for all the possible values of k and N_k the total cost per unit time for the system is calculated. To illustrate the calculations we will take up a particular combination of m_j's and show in detail the calculations of the total cost per unit time for the system for all k s and N_k's for this combination of m_j's.

Let $m_1 = 1, \quad m_2 = 1, \quad m_3 = 1$

set $k = 1$ and $N_k = 0$

DURATION OF MAJOR REPAIR = 0.6

EQUIPMENT	P(I)	AS(I)	BP	AM(I)	TH(J)	AL(I)	ET	D(I)	R(I)
1	130.00	30.00	5.	0.080	0.080	0.700	3.	0.30	50.00
2	120.00	50.00	4.	0.030	0.030	0.800	3.	0.20	100.00
3	110.00	70.00	3.	0.030	0.020	0.900	3.	0.10	140.00

TABLE 4.7: OPTIMAL SOLUTION FOR THE EXAMPLE OF MIXED REPAIR POLICY .

THE OPTIMAL SOLUTION

N*(1) = 1   M*(1) = 1

N*(2) = 1   M*(2) = 1

N*(3) = 3   M*(3) = 1

TOTAL COST PER UNIT TIME = 147.35

OPTIMAL BASIC TIME PERIOD = 3.437

Basic time period $= T = 1.445 + .6 = 2.045$

for equipment (2) $t_2(1, 0) = 1.507 > 2.045 - .6 - .2$

$N_2 = 0$

for equipment (3) $t_3(1, 0) = 1.038 < 2.045 - .6 - .1$

$t_3(2, 0) = 1.829 > 2.045 - .6 - .1$

$N_3 = 1$

Total repair cost per basic cycle

$$= [20 + 30 + 50 + 70] + 10 = 180$$

Total downtime cost per basic cycle

$$= .6 \times 50 + .6 \times 100 + (.6 + .1) \times 140$$

$$= 188$$

Total cost per unit time $= \frac{180 + 188}{2.045} = 179.99$

We set $N_k = 1,2,3,4,5$ and calculate the total cost per unit time for the system. We now, set $k = 2$ and 3 and for both of these, above calculations are repeated for N_k from 0 to 5.

For all possible combinations of m_k s the total cost per unit time for the system is given in Table 4.6. The optimal solution for the system corresponds to $m_1 = 1$, $m_2 = 1$, $m_3 = 1$ and $k = 2$, $N_k = 1$. This optimal solution is given in Table 4.7.

4.5 Joint Repair Policy:

4.5.1 Introduction

In last section we have considered the case where any

TABLE 4.6: TOTAL COST PER UNIT TIME FOR DIFFERENT SETS
SETS OF M(J)'S , K , N(K).

M1	M2	M3	J	VALUES OF N(J)					
				0	1	2	3	4	5
1	1	1	1	179.99	151.80	169.46	195.38	215.78	223.42
			2	176.16	147.35	159.07	185.04	198.50	213.87
			3	210.05	186.46	159.50	147.44	155.31	160.92
1	1	2	1	165.80	180.34	195.00	199.82	206.11	214.76
			2	165.13	179.24	189.03	197.30	199.44	205.58
			3	326.07	229.68	194.61	178.33	170.13	166.00
1	1	3	1	212.42	208.40	215.78	216.86	220.87	227.98
			2	214.92	209.76	212.42	217.02	216.93	221.59
			3	442.09	307.89	258.14	234.25	221.44	214.23
1	2	1	1	172.65	157.13	192.78	242.28	262.25	265.99
			2	274.36	186.40	161.85	166.17	154.44	154.41
			3	188.68	172.05	154.48	154.79	156.66	168.61
1	2	2	1	158.47	185.67	218.31	246.72	252.58	257.33
			2	212.66	157.54	161.24	191.62	202.41	212.35
			3	258.90	200.87	171.20	157.74	162.03	158.39
1	2	3	1	205.08	213.73	239.10	263.76	267.34	270.55
			2	199.68	186.59	189.03	202.05	201.72	202.57
			3	341.34	239.97	202.97	192.70	193.08	187.58
1	3	1	1	172.44	190.87	208.74	238.67	259.12	263.19
			2	385.44	247.91	198.92	188.73	174.10	168.60
			3	189.70	178.23	176.09	184.71	193.94	208.27

1	3	2	1	158.25	219.41	234.27	243.11	249.45	254.53
			2	275.79	191.17	163.46	158.84	155.62	163.31
			3	252.80	191.26	163.44	158.72	159.33	155.85
1	3	3	1	291.87	247.17	255.06	260.15	264.21	267.75
			2	250.53	180.31	174.68	189.74	200.74	214.03
			3	307.75	233.90	197.95	181.28	193.63	178.69
2	1	1	1	307.15	205.38	179.05	162.89	155.04	162.63
			2	181.80	153.30	158.87	200.61	222.52	243.63
			3	205.47	175.49	158.80	152.88	162.70	164.73
2	1	2	1	243.57	175.70	188.51	204.06	215.75	224.69
			2	170.73	185.18	198.82	212.87	223.46	235.34
			3	289.43	223.51	189.59	173.92	168.82	173.62
2	1	3	1	225.86	203.30	213.35	207.93	205.59	215.58
			2	220.57	215.70	222.21	232.59	240.95	251.34
			3	387.13	270.84	228.04	212.64	215.36	208.52
2	2	1	1	253.35	187.80	164.24	157.69	158.08	164.06
			2	247.36	182.03	164.13	163.35	153.88	158.98
			3	184.10	161.09	153.79	160.24	164.05	172.41
2	3	1	1	248.45	186.94	165.85	174.59	189.10	201.45
			2	342.72	227.10	193.90	185.15	181.60	172.26
			3	185.12	167.26	175.39	190.16	201.32	212.08
3	1	1	1	446.05	273.41	212.07	194.12	179.92	181.54
			2	184.18	166.79	193.68	228.44	240.71	251.65
			3	205.35	182.35	167.64	156.56	178.61	181.44
3	1	2	1	333.07	217.70	186.04	172.78	177.84	197.83
			2	173.15	198.67	223.64	240.70	241.64	243.35
			3	295.54	211.07	187.92	181.28	172.83	169.77
3	1	3	1	307.15	212.84	199.10	212.73	228.31	236.80
			2	222.94	229.19	247.03	260.41	259.14	259.36
			3	368.82	277.02	233.05	212.18	203.92	206.61
3	2	1	1	365.35	243.41	189.85	186.31	163.11	167.41
			2	250.63	189.31	166.88	168.05	162.69	170.09
			3	184.40	167.94	162.62	173.92	179.36	189.12
3	3	1	1	338.15	223.80	190.91	183.71	173.40	173.55
			2	329.95	226.09	192.23	191.55	173.54	174.55
			3	185.51	174.11	184.23	203.84	217.23	228.79

number of equipments when repaired together can share a common component of the major repair cost. Under some special circumstances when the set up cost for a major repair is very high, the common component of major repair cost becomes increasingly significant. For such situations, in most of the cases, it may turn out to be profitable to have the major repairs for all the equipments at the same time. In this section we will concentrate on such cases. All the equipments will be forced to have major repairs simultaneously. By performing major repairs on all the equipments simultaneously, we could save on the total fix cost associated with the repairs of each individual equipments.

4.5.2 Assumptions:

In addition to the assumptions of Sec. 4.4, we will have one more assumption.

(9) All the equipments have to undergo major repairs simultaneously. This assumption implies that m_j's have values equal to unity for all the equipments.

4.5.3 Problem Formulation and Solution Procedure:

4.5.3.1 Problem Formulation:

Figure 4.3 shows the sequence of events for joint repair policy.

Let N_j denote the optimal number of minor repairs for

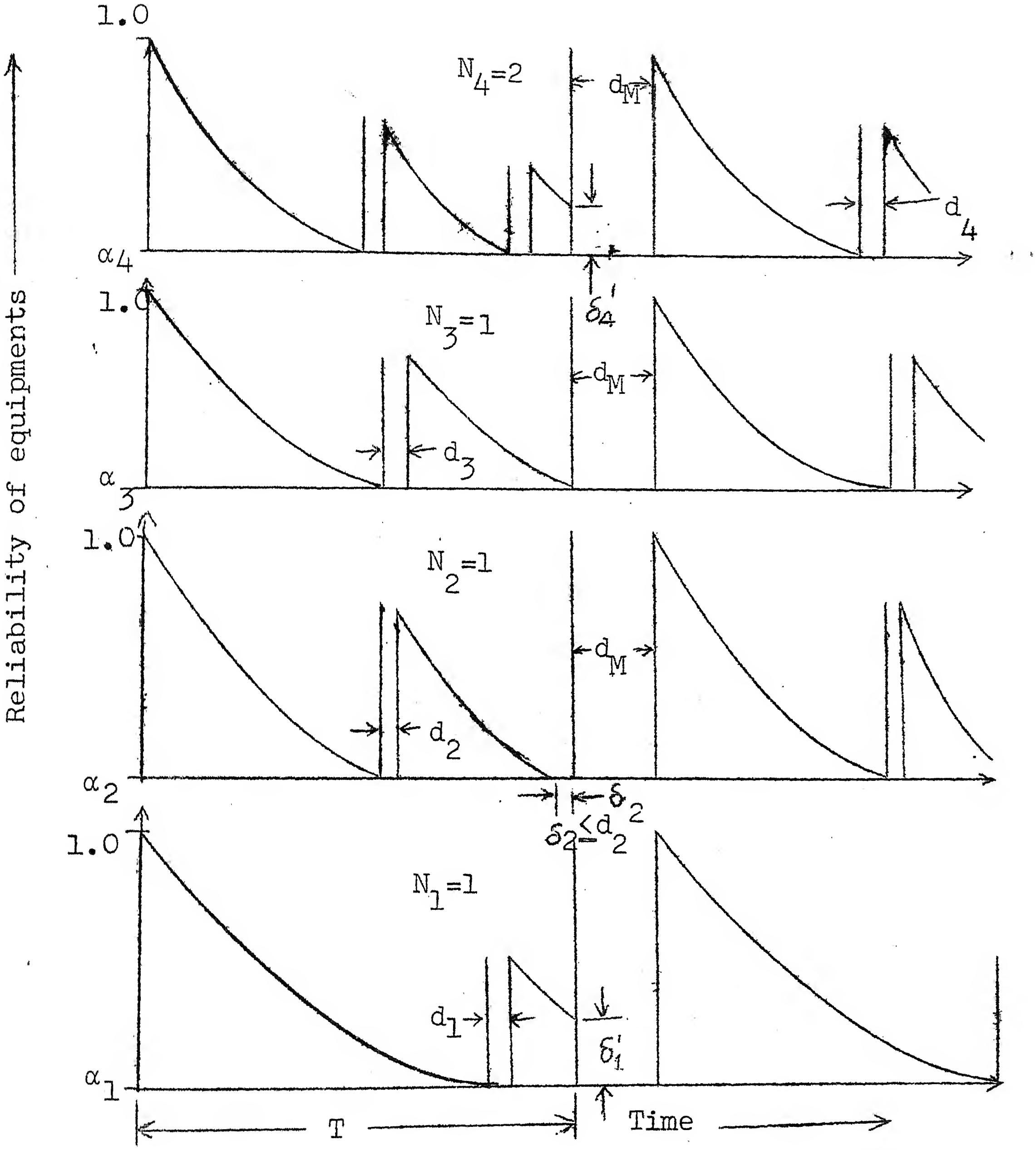


Fig. 4.3: Joint repair policy for service producing system.

equipment j between two consecutive major repairs. It is quite obvious that, for most of the cases, $t_j\ (N_j^* + 1,\ 0)$ may not be exactly equal for all the equipments. As a result many of the equipments will have to wait for some time before the major repairs for all the equipments can take place simultaneously. Let δ_j denote the time for which the equipment j has to remain in nonoperating condition before the major repair is taken up. δ_j can be given in terms of $t_j\ (N_j^* + 1,\ 0)$, and d_M as

$$\delta_j = \max\ (0,\ (T - d_M) - t_j\ (N_j^* + 1,\ 0))$$

For the system depicted in Fig. 4.3, we see that the equipment two waits for δ_2 time units before the major repair starts. On the other hand for equipment one the major repair begins even before the reliability level reaches the LARL. These situations can be improved. We will consider these, in brief, in Sec. 4.6. In present section, like the previous section, we will not make any compromization between the lowest acceptable reliability level and the downtime costs. We will stick to rule one and rule two mentioned in Sec. 4.4.3.1.

Downtime for the equipment j, between two major repairs can be written as:

$$DT_j = N_j\ d_j + d_M + \max\ (0,\ m_j T - d_M - t_j\ (N_j + 1,\ 0))$$

The downtime cost per unit time for equipment j is given by,

$$DTC_j = \frac{DT_j}{T_j} \cdot R_j$$

The repair cost for all the equipments per unit time is given as:

$$RC = [A + \sum_{j=1}^{M} AS_j + N_j P_j] \frac{1}{T}$$

The total cost per unit time for the system can now be written as summation of the above two cost components.

$$TC = \sum_{j=1}^{M} DTC_j + RC$$

$$= \frac{1}{T} [A + \sum_{j=1}^{M} [P_j N_j + AS_j + R_j \cdot DT_j]] \qquad (4.10)$$

4.5.3.2 Solution Procedure:

It has been seen in Sec. 4.3.3 that at least for one of the equipments δ_j has to be zero in the optimal solution. The value of optimal cycle time can, therefore, be expressed as,

$$T = x + d_M$$

where, $x \in \{ t_j (N_j + 1, 0)/j = 1,\ldots M \text{ and } N_j = 0,\ldots,i_{j\,max}\}$

(4.11)

Here $i_{j\,max}$ is the empirical upper limit on the number of minor repairs between two consecutive major repairs. The value of $i_{j\,max}$ is obtained in the same way as it was done in Sec. 4.4.3.2. Proceeding on the parallel lines of the solution procedure for mixed repair case, we first select the equipment one as the equipment for which $t_j (N_j^* + 1, 0)$ is exactly equal to $T - d_M$. Now we fix different values to N_1 and calculate, for each such value of N_1, the values of T, N_j s for other

equipments and the total cost per unit time for the system. The same procedure can be repeated by setting $t_j\ (N_j^* + 1, 0)$ exactly equal to $T - d_M$ for equipment two, three and others. Now the solution which corresponds to the minimum total cost for the system can be selected as the optimum solution for our joint repair scheduling policy. The procedure can be stated in following steps:

Step 1: Calculate G_j's and $T_j(i, 0)$'s for all the equipments and for i given by $(0 < i \leq i_{max})$.

Step 2: Set $k = 1$, $N_k = 0$.

Step 3: Obtain the basic time period for the system as:

$$T = t_k\ (N_k + 1, 0) + d_M$$

The values of N_j's for all other equipments are given by following inequalities,

$$t_j\ (N_j, 0) \leq T - d_M - d_j$$

$$t_j\ (N_j + 1, 0) > T - d_M - d_j$$

Calculate the total cost per unit time from Eqn.(4.10).

Step 4: $N_k = N_k + 1$, If N_k is not greater than $i_{k\ max}$, go to Step 3, otherwise $k = k+1$, If k exceeds M (the number of equipments) then go to Step 5, otherwise, $N_k = 0$, go to Step 3.

Step 5: Select that set of decision variables which gives minimum cost per unit time.

4.5.4 Numerical Example:

We consider the following example, to illustrate the solution procedure for obtain the optimum solution, for joint repair policy.

Let us consider the same system which we considered in Sec. 4.4.4. The values of G_j and t_j (i, 0) for all the equipments and all the i's are already obtained. We will solve the above problem under the constraint that all the major repairs have to take place simultaneously.

Let us first set $k = 1$, $N_k = 1$ and $T = t_j(N_k + 1, 0) + d_M$

$$T = 2.9 + .6 = 3.5$$

for equipment two,

$$t_2(1, 0) = 1.507 \leq 3.5 - .6 - .2$$

$$t_2(2, 0) = 2.837 > 3.5 - .6 - .2$$

therefore, $N_2 = 1$ and similarly $N_3 = 4$.

The total cost per unit time for the system can be calculated as.

$$TC = RC + \sum_{j=1}^{M} DTC_j$$

where,

$$RC = \text{repair cost per unit time}$$

$$= \frac{1}{3.5}[20 + 30 + 50 + 70 + 30 + 20 + 10 \times 4]$$

$$= 74.286$$

the downtime costs per unit time are given as,

$$DTC_1 = \frac{1}{3.5} [(.6 + .3) 50] = 12.86$$

$$DTC_2 = \frac{1}{3.5} [(.6 + .2 + .063) 100] = 24.66$$

$$DTC_3 = \frac{1}{3.5} [(.6 + .1 \times 4) 140] = 40.0$$

Total downtime cost for the system

$$= \sum_{j=1}^{M} DTC_j = 12.86 + 24.66 + 40 = 77.52$$

Total cost for the system (per unit time)

$$= 77.52 + 74.286 = 151.8$$

Above steps are now repeated for different values of N_k's for different k's. The corresponding decision variables and the objective functions are given in Table 4.8.

The optimal value of total cost per unit time corresponds to $k = 2$ and $N_k = 1$. This optimal solution is given in Table 4.9.

4.6 Scope for Future Studies:

For future studies on scheduling of repairs for service producing systems, following improvements can be incorporated over and above the present study.

(1) The general form of a problem of obtaining the optimal

TABLE 4.8: VALUES OF BASIC TIME PERIOD AND THE TOTAL COST CORRESPONDING TO VARIOUS K AND N(K).

N1	N2	N3	K	T.COST	TBASIC
0	0	1	1	179.99	2.045
1	1	4	1	151.80	3.500
2	3	10	1	159.46	4.725
3	4	20	1	195.38	5.764
4	5	N(3) EXCEEDS I3MAX.			
5	7	N(3) EXCEEDS I3MAX.			
0	0	1	2	176.16	2.107
1	1	3	2	147.35	3.437
2	2	8	2	159.07	4.484
3	3	16	2	185.04	5.320
3	4	22	2	198.50	5.997
4	5	N(3) EXCEEDS I3MAX.			
0	0	0	3	210.05	1.638
1	1	1	3	186.46	2.429
1	1	2	3	159.50	2.991
1	1	3	3	147.44	3.398
1	2	4	3	155.31	3.703
2	2	5	3	150.92	3.940

TABLE 4.9: OPTIMAL SOLUTION
(EXAMPLE OF JOINT REPAIR POLICY)

N*(1) = 1

N*(2) = 1

N*(3) = 3

TOTAL COST PER UNIT TIME = 147.35

OPTIMAL BASIC TIME PERIOD = 3.437

repair schedule for a service producing system can be given as follows:

Objective: To minimize total cost per unit time for the system.

Subject to: Reliability constraints and availability constraints.

The objective function includes only the cost due to repair. The reliability constraints have already been dealt with in detail in present chapter and the availability constraints are usually of the form:

$$\text{Availability for equipment } j \geq x_j \quad \forall\ j$$

where x_j is allowable availability level for equipment j.

Total service produced by a particular equipment per unit time can be obtained directly by knowing the fraction of time for which that equipment was available. In many practical situations this service level may not be allowed to fall below a particular level and therefore, it becomes mandatory to treat availability of equipments, as constraints on the system.

The general constrained problem discussed above can be made unconstrained by incorporating both the availability constraints and the reliability constraints, in the objective function. To obtain this we may introduce additional costs

associated with the deviation of reliability of a equipment from one and the deviation of downtime from zero. In fact, in our present study we have incorporated the downtime constraints in the objective function itself by assigning additional costs due to downtime duration. The cost due to deviation of reliability from one may be termed as unreliability cost. Unreability cost at any instant is given by:

$$\text{Unreliability cost} = f\,[1 - Re_j(t)]$$

The unreliability cost for equipment j between i-th and (i+1)-th minor repair is given by:

$$URC_j\ (i,\ i+1) = \int_{t_j(i,1)}^{t_j(i+1,0)} f(1 - Re_j(t)).dt$$

(2) Our optimal solution for a system may sometimes require one of its equipments to undergo a major repair even before it touches its LARL. Under such circumstances it is quite possible to increase the LARL for that equipment by certain amount without changing the time period, the downtime cost per unit time and the number of minor repairs between two consecutive major repairs. This increase in LARL is associated with decrease in $t_j(i,\ 0)$'s. Best LARL corresponds to that value for which $t_j\ (N_j^* + 1,\ 0)$ is equal to $(m_jT - d_M)$ for that equipment.

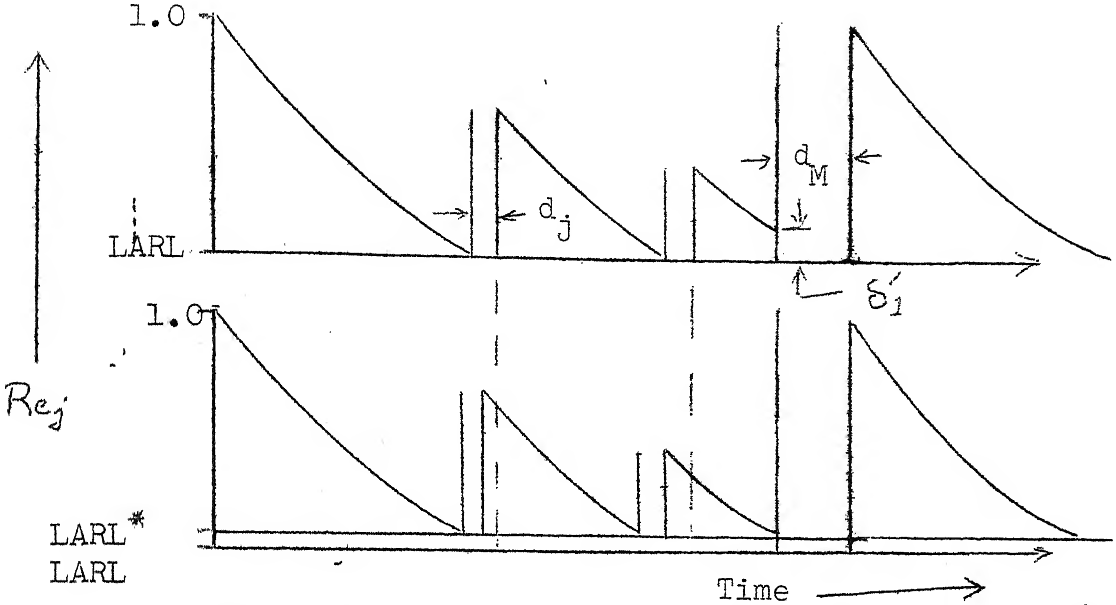


Fig.4.4: Improving LARL(without additional cost.)

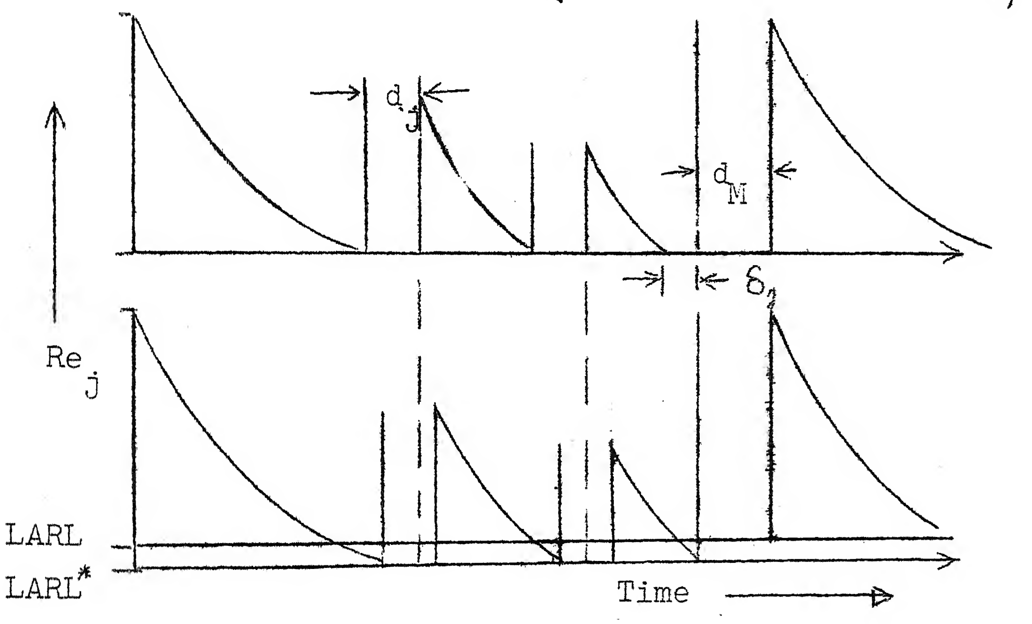


Fig. 4.5: Reduring downtime cost by decreasing LARL.

(3) Our optimum solution for a system may sometimes require one or more of its equipments to wait for some time before it can undergo a common major repair. By decreasing the LARL for such equipments we may reduce the downtime cost. Any decrease in LARL, in turn, indicates a worse performance for such equipments. This proposed decrease in LARL can, therefore, be obtained by striking a balance between the resulting decrease in downtime cost and the performance of the equipment.

For future studies concerning scheduling of repairs for service producing systems we can incorporate one or more of the above modifications.

CHAPTER V

STOCHASTIC STUDIES

5.1 Introduction:

In Chapter II and III we have dealt with only those systems which are completely deterministic in nature. We had assumed the planning horizon to be infinite. As a result of this deterministic nature of the system and the infinite planning horizon the repair schedules were cyclic in nature. It was also assumed that the repair durations, for both minor and major repairs, remain constant irrespective of the age of the equipment. In the present chapter we will assume the repair durations to be probabilistic in nature, however, the distribution of these repair durations are assumed to be known with known parameters. This nondeterministic nature of the repair duration results in acyclic repair schedules. In Chapter II and III we had assumed that no failures due to chance causes take place and therefore only the preventive repair schedules were considered. In this chapter, we will assume failures due to chance causes also. These failures are assumed to be stochastic in nature with known distribution and distribution parameters. Occurrance of such failures demands for immediate corrective action. Thus along with the

preventive maintenance, which is decided beforehand, the equipments, sometimes, have to undergo corrective maintenance also, whenever any such need arises. Unlike Chapter II and III, we assume that we have only one repair facility. Selection of a equipment for repair, whenever two or more equipments are waiting in queue, is done on first come first serve (FCFS) basis along with a particular priority rule. This priority rule will be discussed in detail when we talk about system repair policy.

5.2 System Description:

We consider a goods producing system which comprises of several equipments. The operating characteristics of these equipments are totally independent of the operating characteristics of other equipments. When the production cost per unit for any equipment goes up, then that equipment can undergo either a minor repair or a major repair. The repair distributions for all these equipments follow particular distributions with known distribution parameters. Any equipment when in operation can be a subject to chance failures also. These failures are treated as most urgent. The repair durations for such failures are also assumed to follow a particular known distribution.

The concept of time period for a particular equipment, or basic time period for the whole system has got no meaning

for the present situation, because, all the repair durations are no longer constant as they used to be in Chapters II and III. However, we will assume that the effect of these repairs on the production cost per good unit is still the same, and this is not function of the repair duration. This is equivalent to saying that improvement factor for any equipment due to a repair is not a function of the repair duration. We will also assume that for any equipment the operation duration between any two consecutive repairs is always same, that is the concept of basic operation duration still holds good. Once again due to non deterministic nature of the repair durations, the term m_j (number of major repairs of equipment r for each major repair of equipment j) has got no meaning. For our present case repair schedule is defined by the number of minor repairs between two consecutive major repairs and the basic operation duration for all the equipments of the system.

When a major repair for any equipment j is scheduled, then, all those equipments for which the scheduled major repairs are 'not very for away,' also have their major repairs with this equipment j. The advancement is scheduled major repair timings is done in order to save the common set-up cost for the major repairs. At a particular instant, all those equipments, for which the next scheduled major repair is 'not very far' away, are said to be in 'can be repaired' state. To

scientifically define the 'can be repaired' state we select a number i_j between 0 and N_j. At any instant of time, all those equipments, for which either $N_j - i_j$ or more minor repairs have taken place after the most recent major repair, are said to have the 'can be repaired' state.

When more than one equipment is waiting in queue for getting repaired, we use following priority rule along with the first come first service rule for selecting the equipment which should get repaired before others.

Priority Rule: Lower priority number indicates higher priority preference.

Table 5.1: Priority Index

State	Priority Number
1	Breakdown repair
2	Major repair
3	Major repair of the equipment which was in can be repaired state.
4	Minor repair

5.3 System Performance:

5.3.1 Performance Measures

System performance refers to a sequence of states that a system assumes over a specified time interval. The

reason why we want to analyse a system, is to understand how changes occur in the system when certain parameters are varied in a prespecified manner. Our basic intentions are to predict the changes and then control them so as to yield the best system performance. Following performance measures are of importance for our present system.

Performance measures:

1. Cost per unit time for the whole system.
2. Availability of each individual equipment expressed as fraction of time for which that equipment remains in operating state.
3. Availability of all the equipments simultaneously, that is, the fraction of time for which all the equipments are in operating state at the same time.
4. Utilization of the repair facility.

In general a system can have many performance measures. For such systems we average out these measures in some way to (for example by assigning appropriate waitages) provide a single performance measure. For our purpose, very much like Chapter II and III, we will use cost per unit time for the whole system, as a performance measure.

5.3.2 Performance Optimization:

The ideal objective for a computer simulation is to optimize the system performance. As we have already said that

our system performance will be measured by cost per unit time, we would like to minimize this. While describing the system, we had mentioned that a repair schedule is defined by the number of minor repairs between two consecutive major repairs and the basic operation duration between any two consecutive repairs for all the equipments. Performance optimization, therefore, involves controlling the decision variables N_j's and t_j's (used in their normal sense) so that the best system performance can be realized. Besides minimizing the total cost for the system, it may be considered necessary to make sure that the availability of any equipment does not fall below a predetermined level for that equipment, but in our present analysis we are not considering those constraints. We will also not consider the utilization factor for the repair facilities as one of the constraints or the number of repair facilities as one of the decision variables, for the system. To simplify the situation we will deal with systems having only one repair facility.

5.4 System States and Events:

Our system contains two entities, the equipments and the repair facility. State of the system is defined as a combination of the states of each of the equipments and the repair facility. A equipment can have any of the states mentioned in Table 5.2.

Table 5.2: Equipment States

State Numbers	State Descriptions
~~1.~~	~~Failed (Breakdown has occurred)~~
2.	Waiting for a minor repair
3.	Waiting for a major repair
4.	Waiting for a breakdown repair
5.	Getting repaired
6.	Can be repaired
7.	Operating well

Except for the 'can be repaired' state all other state descriptions are self explanatory. The 'can be repaired' state has been defined in Sec. 5.2. The repair facilities can have only two states, it can be either busy or idle. The busy state is denoted by 1 and the idle state is denoted by 0. Change in the state of any of the entities of the system can take place only when one or more events take place. These events for all individual equipments are listed in Table 5.3.

Table 5.3: Event Description

Event Number	Event Description
1.	Scheduling of a minor repair
2.	Scheduling of a major repair
3.	Scheduling of a breakdown repair

Table 5.3: continued

4.	Beginning of a minor repair
5.	Beginning of a major repair
6.	Beginning of a breakdown repair
7.	End of a minor repair
8.	End of a major repair
9.	End of a breakdown repair

The description of events is presented below.

1. Scheduling of a minor repair:

Whenever a minor repair gets scheduled, we check whether the repair facility is idle or not. If the repair facility is idle we can immediately schedule the beginning of this minor repair, otherwise this equipment will be placed in queue of equipments which are waiting for repair. If the equipment is put in queue then the state variables gets a value of equal to two, indicating that the equipment is waiting for minor repair and the priority index takes the value of 4 (See Table 5.1.). We also calculate the production cost since the end of recent repair and update the time for which the equipment has operated since the beginning of the simulation.

2. Scheduling a Major Repair:

Very much like the minor repair, either the equipment gets put in the queue and waits for the repair facility to get free or the beginning of this major repair gets scheduled immediately, depending on whether the repair facility is busy or idle. The production cost since the end of last repair along with the total time for which the system has operated so far, gets updated. If the equipment is put in the waiting queue then the state variable and priority index are assigned appropriate values depending on whether this major repair was scheduled independently or it was scheduled because of some other equipment which was having its major repair scheduled independently, when this equipment was in 'can be repaired' state.

3. Scheduling of a Breakdown repair:

This event is same as scheduling of a minor repair except for the fact that when the equipment is placed in waiting queue then the priority index gets a value of one and the state variable gets a value of four indicating that the equipment is waiting for a breakdown repair.

4. Beginning of a Minor repair:

When a minor repair for a equipment begins, then the idle time for the repair facility and the waiting time

for this equipment gets updated. If the machine has been in idle mode, then it is put in busy mode and the end of this minor repair is scheduled. The state variable is set at 5 indicating that the equipment is getting repaired. The total cost is also updated by adding the downtime cost for the period for which the equipment had waited in the queue.

5. Beginning of the Major repair:

Similar to event four here also we update the idle time for the repair facility, the waiting time for the equipment in queue and the total cost for the system. The repair facility is set in busy mode and the end of this major repair is scheduled. If the machine had been in idle mode or it had performed a repair operation which was not major, just before this major repair is taken up, then the set-up cost for major repair is also added in the total cost for the system.

When a major repair gets started then we look for all those equipments which are in can be repaired state and schedule a major repair for all of them. All these equipments have priority index number as 3.

6. Beginning of a Breakdown repair:

When a breakdown repair starts then the event which is to be scheduled next for that equipment, is the end of this breakdown repair. Everything else remains same as it was in case of beginning of a minor repair.

7. End of Minor repair:

At the end of a minor repair we update the total time for which the equipment had been out of operation. The next repair for this equipment is scheduled. This repair can be of either of the three kinds of repairs, minor, major, or breakdown, depending on the value of N_j's and the time for the next breakdown. The state variable is set at one, two or three when a minor, major or breakdown repair is scheduled respectively. The number of minor repairs that have taken place after the most recent major repair is updated. If this number is either equal to or more than $N_j - i_j$ (terms have their usual meaning) then the state variable is set equal to six indicating that the equipment is in can be repaired state. Update the total cost of the system by adding the downtime cost, for the period for which the equipment was getting repaired, plus the repair cost.

8. End of Major repair:

Similar to event seven we update the total cost for the system and the downtime duration for the equipment. The number of minor repairs that have taken place after the most recent major repair is set equal to zero. Scheduling of next event for this equipment is done in the same way as it was done in case of event seven. The state variable is given a value of one, two or three depending upon the kind of repair that is

scheduled for that equipment. If N_j is equal to i_j for this equipment then the state variable is set at six, indicating that the equipment is in can be repaired set, otherwise the state variable is set of seven, indicating that the equipment is working well.

9. End of a Breakdown Repair:

Everything remains exactly same as it was in case of event- eight, except the updating of the total cost. The cost of breakdown repair along with the downtime cost for the equipment, for the time for which it was getting repaired, is added in the total cost of the system.

Whenever an event ends, the next event for the system is selected. After every event the termination criteria is checked and if it gets satisfied the simulation is stoped. Flow chart for the problem is given in Fig. 5.1(a) - (d). All the variables used in it are given in Table 5.11.

5.5 Data Generation:

Data about the following performance indices are generated in order to understand and control the effect of decision variables on the system performance.

1. Waiting Time:

It is always preferred to have lower waiting time for every equipment, so that the production rate is not hampered,

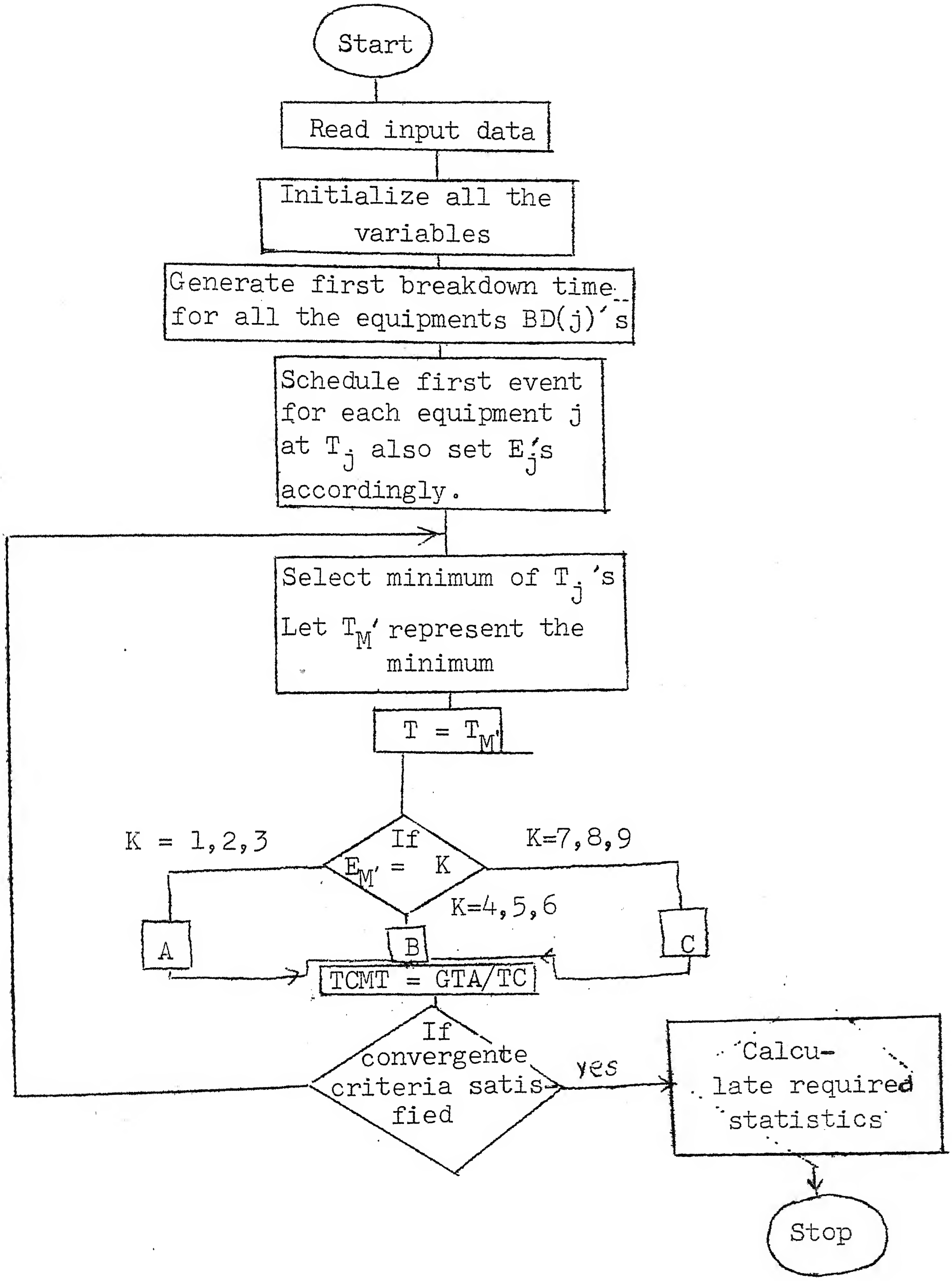


Fig. 5.1(a): Simulation Flowchart.

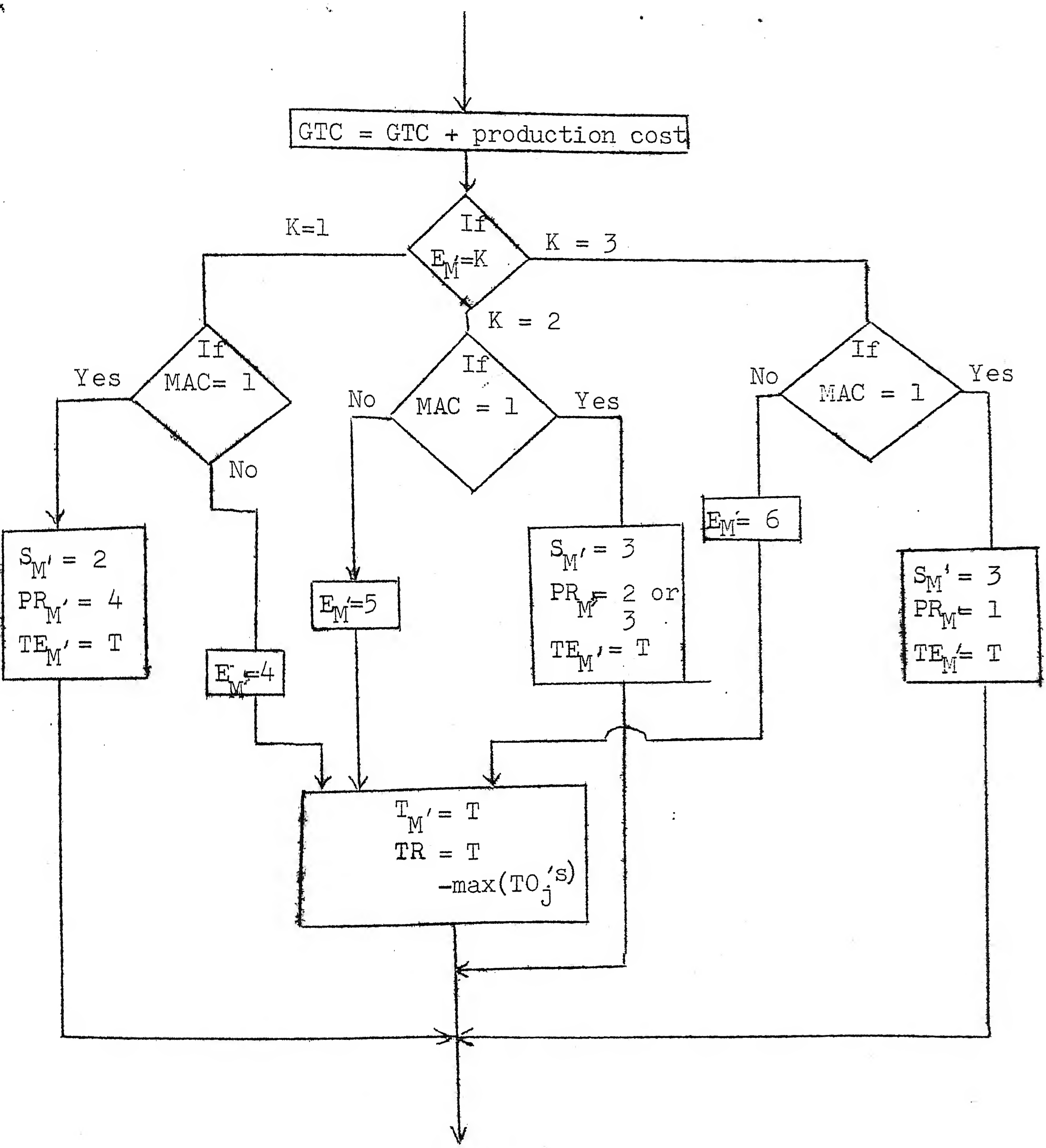


Fig. 5.1(b)+ Block A in 5.1(a).

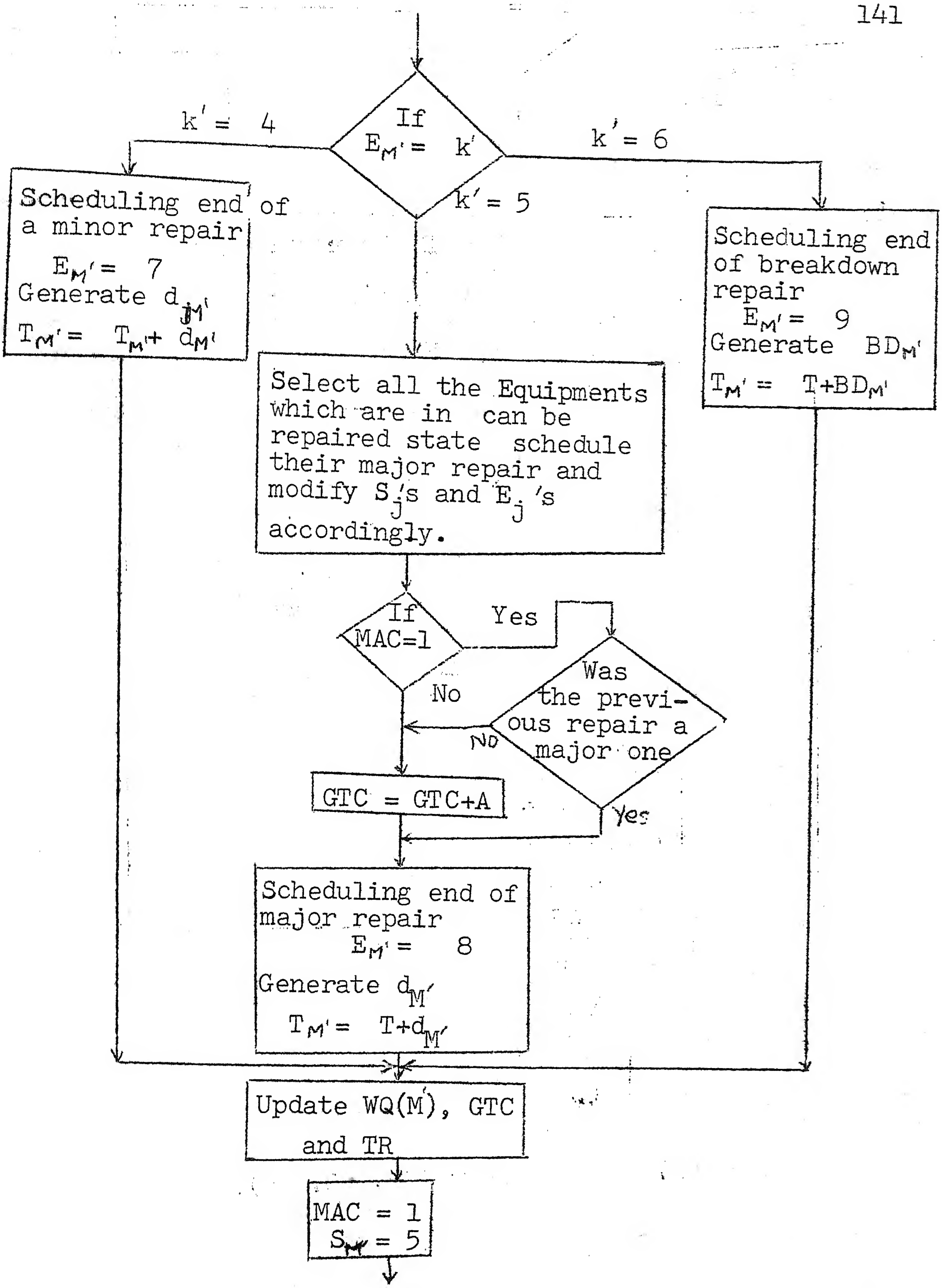


Fig. 5.1(c) : Block B of Fig. 5.1(a).

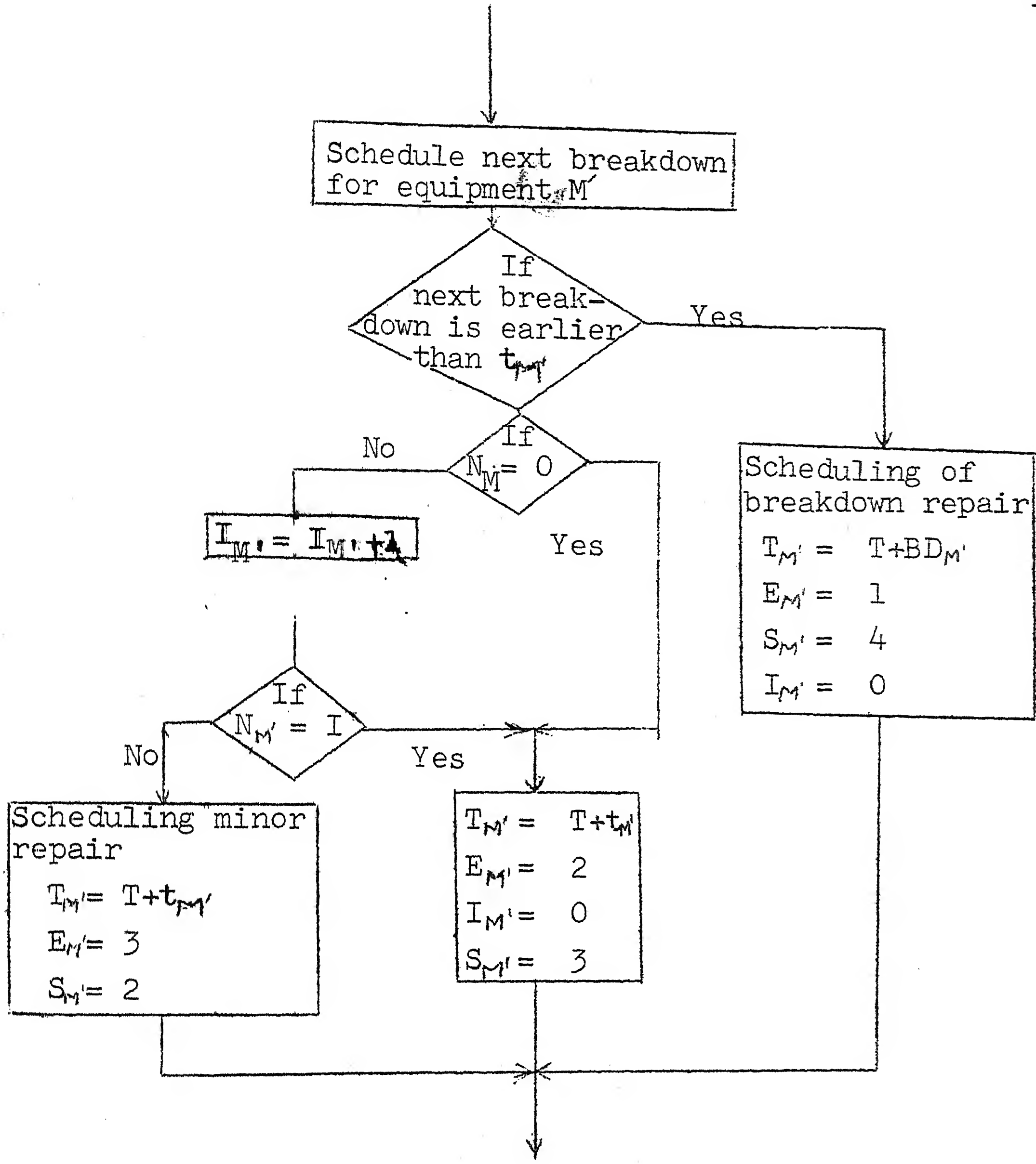


Fig. 5.1(d): Block C in Fig. 5.1(a).

seriously, because of the equipments which are not in operating state. Throughout the simulation we update the total time for which a equipment remains in non operating state. This directly gives the fraction of time for which the equipment is not available for production. Some idea about the total production rate can also be derived from here. We also update the fractional waiting time, that is, the fraction of the total simulation run time which a equipment spends in the queue before it gets a chance to get repaired.

2. Idle Time for the Repair Facility:

We update the total time for which the repair facility remains in the idle state throughout the simulation run. Since we are dealing with only those systems which have fixed number of repair facilities (only one for our present case), the study of fractional nonoperating time for the repair facility will not be of much help. If we had assumed variable number of repair facilities then this could have been useful for obtaining the optimal number of repair facilities for the system.

3. Total Cost Per Unit Time:

Whenever any event takes place, we update the total cost for the system. When a repair is scheduled, we add the production cost for that equipment from the end of most recent repair till the time, under consideration. At the beginning of

any of the repairs the downtime cost for the period for which the equipment had waited before getting repaired is calculated and added to the total cost. The setup cost for a major repair is also added to the total cost when no major repair had taken place just before this major repair is taken up. At the end of a repair the repair cost and the downtime cost for the time, during which the equipment was having repair, is added to the total cost for the system. The total cost per unit time is obtained by dividing the total cost during the simulation run by simulation run time. When the total cost per unit time is minimized, then the corresponding decision variables are the optimal decision variables for our system.

5.6 Termination Criteria:

Sometimes, in simulation studies, the termination criterion is either expressed in terms of elapsed number of simulated time units or in terms of number of completed jobs. In most of the cases some statistical termination criteria are also employed to ascertain the results, obtained after simulation with certain degree of confidence in them and to make sure that the system has really attained a steady state. These statistical controls can be employed accurately only when the exact distribution pattern and parameters are known for all the variable, of which the statistical stability is being examined for termination of simulation. For our present

simulation purpose we have fixed the minimum number of events that should take place, as one of the termination criterion. We are also examining the statistical stability of the total cost per unit time. Since the exact distribution for the total cost per unit time is not known we have approximated it to a normal distribution, the distribution parameters of which are estimated from the data generated by simulation itself. When both of the termination criteria get satisfied the simulation gets terminated.

5.7 Optimization

It is obvious that because of the highly complicated nondeterministic nature of the problem, it is very difficult to use any analytical method for obtaining the optimum or near-optimal solution for this problem. Since simulating any such system is very time consuming, we would prefer to reduce our interval of uncertainity for all the decision variables, which are needed to be explored for obtaining the optimal solution. To do so, we solve a parallel deterministic joint repair scheduling problem. For this new problem the repair durations for every equipment is equal to the mean of corresponding repair duration distribution. No breakdowns can be considered where we are dealing with the deterministic case of joint repair policy.

In absence of failures due to chance causes it was seen in Chapter II and III that the objective function is unimodular in t_j's (the basic operation durations) for given set of N_j's. We will assume here that for all the practical purposes this unimodularity condition still holds good even in presence of chance-failures. Now if the results of simulation show that the optimal values t_j's as obtained after exploring all the possible sets of t_j's in the intervals, selected in the vicinity of t_j^{*}'s, does not turn out to be one of the extreme points of these intervals, then this optimal solution can safely be taken as global optimum solution for given set of N_j's. Here t_j's are the optimal solutions corresponding to the joint repair scheduling problem, mentioned above.

We select the initial expected interval for the optimum values of all the decision variables in such a way, that these intervals sprawl, equally on either side of the optimum value of the corresponding variable, in the solution of the parallel-joint-repair-scheduling problem, discussed above. We select a particular set of N_j's (obviously in the intervals mentioned above). We then find out some expected value for the optimum t_1 by exploring through the initial expected interval for the optimum value of t_1 , keeping all other basic operation durations set at t_j^{*}'s. Some unimodular searching technique is used for this purpose the size of the initial intervals can be

decided intuitively and we now fix the basic operation duration for equipment one at the value obtained after this search and explore through the initial interval of expected optimum value of t_2, to obtain an approximation for the optimum value of t_2. This process is then continued till we get the approximate optimum solution for the values of all t_j's. We now again repeat this cycle starting from first equipment till the last equipment after intuitively selecting new initial intervals for all the t_j's. These new intervals are in general much smaller in size as compared to the previous intervals. Once this second scan gets completed, it can be expected that the so obtained approximate optimum values t_j's are very close to the actual optimum values, which correspond to the realization of best performance index.

We now select another set of N_j's and repeat the above process for obtaining the optimum values of basic operation durations. All the possible combinations of N_j's in the selected intervals are tried the global optimal solution is then obtained by selecting the best out of these solutions.

This procedure can be described in following steps.

Step 1: Assume that the failures due to chance causes do not occur and the repair durations for all the repairs are equal to the mean of the corresponding repair distributions. Solve this problem as a deterministic major repair scheduling problem.

Let the solution be denoted as N_j^* s and t_j^* s.

Step 2: The range of N_j s, which is to be explored for obtaining the optimal solution can usually be taken as $N_j^* \pm k_j$ where k_j is an intuitive constant integer. The initial expected intervals for the optimum value of t_j's can be selected intuitively in such a way that the corresponding t_j^*s, obtained in step 1, fall nearly in the middle of that interval.

Step 3: Select a set of N_j's such that,

$N_j^* - k_j \leq N_j \leq N_j^* + k_j$

Step 4: Set $t_j = t_j^*$ $\forall$ j except j = 1, set r = 1.

Step 5: (a) Do Febonacci search (or any other unidirectional search) for obtaining some approximate value of the optimum t_r.

(b) Set t_j equal to the value obtained in Step 5(a).

Step 6: Repeat Step 5 for all the values of r from 2 to M, where M is the number of equipments.

Step 7: Set r = 1 again and intuitively select smaller initial interval for all variables such that the corresponding approximate optimum value obtained in Step 5 fall nearly in the middle of the intervals. Perform Step 5 and Step 6.

Step 8: Perform Step 4 to 7 for all possible set of N_j's and select that set of N_j's and the corresponding t_j's, which correspond to the minimum total cost per unit time.

5.8 Numerical Example:

To illustrate the solution procedure we consider a three equipment system with one repair facility. The related parameters are given in Table 5.4. We first solve an approximate parallel joint repair scheduling problem with:

$$d_M = .5 + .4 + .3 = 1.2$$

The optimal N_j's and t_j's for this parallel problem are given in Table 5.5. Since for solving this parallel problem we had assumed ample number of repair facilities as compared to, one for the original problem, the solution of original problem will have large operation durations than t_j^*s. Keeping this in view we set initial intervals of uncertainity for our decision variables as:

$$0 \leq N_j \leq 2 \qquad \forall\, j$$
$$1.277 \leq t_1 \leq 9.277$$
$$0 \leq t_2 \leq 7.438$$
$$0 \leq t_3 \leq 7.538$$

We set $N_1 = 0$, $N_2 = 0$, $N_3 = 0$, $t_2 = 1.438$ and $t_3 = 1.538$ and make a Febonacci search for t_1 in above given interval. Table 5.6 gives the result of this search.

TABLE 5.4: INPUT DATA FOR SIMULATION PROGRAM.

PARAMETERS	EQUIPMENT 1	EQUIPMENT 2	EQUIPMENT 3
MINOR REPAIR DURATIONS MEAN	0.3	0.4	0.2
MAJOR REPAIR DURATIONS MEAN	0.5	0.4	0.3
BREAKDOWN REPAIR DURATIONS MEAN	0.5	0.4	0.3
MINOR REPAIR COST	30.0	30.0	30.0
MAJOR REPAIR COST	40.0	40.0	60.0
BREAKDOWN REPAIR COST	40.0	30.0	20.0
DOWNTIME COST RATE	60.0	100.0	140.0
PRODUCTION COST FUNCTION	5+2*T**2	5+4*T**2	5+6*T**2
IMPROVEMENT FACTOR	5.0	4.0	3.0
BREAKDOWN RATE	60.0	80.0	100.0

TABLE 5.5: SOLUTION OF APPROXIMATE PROBLEM

N(1) = 0	TS(1) = 3.277
N(2) = 1	TS(2) = 1.438
N(3) = 1	TS(3) = 1.538
TOTAL COST = 193.55	

Table 5.6: Febonacci Search for Optimal t_1 with $t_j = t_j^* \;\forall\; j$ except $j = 1$ and $N_j = N_j^* \;\forall\; j$.

t_1	TC_1
1.277	198.97
4.354	131.76
6.200	131.76
9.277	132.10
7.43	131.71
8.043	131.83
6.815	131.46

The final interval of uncertainity is from 6.2 to 7.43. We select the point 6.815, which corresponds to minimum value of TC and set $t_1 = 6.815$. We repeat similar ~~search~~ for t_2 and t_3. This search yields:

$$t_2 = 4.58 \text{ and } t_3 = 3.48$$

We again repeat this Febonacci search in intervals sprawling over one time unit on either side of these values. The result of this search is given below:

$$t_1 = 6.28, \quad t_2 = 3.58 \quad t_3 = 3.48$$

The objective function value for this solution is 131.21. Table 5.7 gives the details of this Febonacci search .

We now repeat the entire procedure with other combinations of N_j's. Corresponding optimal solutions are given in Table 5.8. Finally we select the best combination of N_j's as the optimal solution. This solution is given in Table 5.9.

So far we had assumed that, whenever, for an equipment, N_j minor repairs have taken place and the equipment is now waiting for next major repair, then this equipment was said to be in 'can be repaired' state . We repeat the entire procedure for the case when no equipment is ever supposed to be in 'can be repaired' state. The optimal solution for this problem is given in Table 5.10.

5.9 Comment:

A wide discrepancy can be observed between the solutions obtained from simulation and the parallel-joint-repair-scheduling problem. Following arguments can be presented in order to explain this discrepancy.

Number of Repair Facilities:

Our joint repair scheduling policy was developed under the assumption that the number of repair facilities are large enough to make sure that none of the equipments have to wait for getting repaired. For our simulation studies we have assumed only one repair facility and, therefore, many of the equipments may have to wait for their turn when some other

TABLE 5.8: OPTIMAL OPERATION DURATIONS AND TOTAL COST FOR VARIOUS SET OF N(J)'S.

MINOR REPAIR INTERVAL			NO. OF MINOR REPAIRS PER MAJOR REPAIR			UTILITY FACTOR FOR EQUIPMENTS			UTILITY FACTOR FOR MACHINE	SIMULATION	STND DVTN	AVRG COST PER UNIT
6.3	4.8	3.6	0	0	0	0.884	0.910	0.929	0.142	1397.7	22.8	131.21
7.0	3.8	1.8	0	0	1	0.900	0.924	0.941	0.118	859.6	44.6	140.92
6.9	4.2	2.5	0	0	2	0.889	0.929	0.906	0.156	517.1	15.2	136.33
6.0	4.4	3.5	0	1	0	0.882	0.905	0.930	0.144	1369.7	24.2	131.73
6.1	2.8	2.0	0	1	1	0.902	0.841	0.898	0.240	898.7	69.3	169.19
4.4	2.0	2.7	0	1	2	0.903	0.850	0.905	0.224	284.8	43.9	148.59
4.6	2.5	4.2	0	2	0	0.907	0.845	0.945	0.199	238.5	13.5	135.27
4.4	2.4	3.2	0	2	1	0.892	0.827	0.895	0.252	265.7	41.1	151.33
5.2	2.8	2.9	0	2	2	0.918	0.876	0.901	0.186	724.7	48.4	137.17
5.9	4.0	3.5	1	0	0	0.877	0.905	0.931	0.147	1281.7	24.8	131.56
5.8	3.0	3.9	1	0	1	0.869	0.893	0.925	0.158	1559.3	25.7	133.29
5.0	2.7	3.5	1	0	2	0.858	0.886	0.929	0.195	177.1	19.8	136.10
5.0	1.6	3.9	1	1	0	0.876	0.819	0.939	0.247	187.0	20.0	146.68
5.3	1.5	4.6	1	1	1	0.884	0.803	0.933	0.245	589.6	49.0	149.17
5.0	1.3	3.4	1	1	2	0.886	0.792	0.922	0.278	144.3	40.4	154.18
5.1	2.1	3.8	1	2	0	0.883	0.848	0.929	0.215	171.3	18.3	136.51
5.7	1.3	2.2	1	2	1	0.919	0.792	0.900	0.291	178.5	81.3	175.56
5.0	1.2	2.6	1	2	2	0.916	0.767	0.905	0.293	264.8	65.0	161.38
3.8	3.0	3.6	2	0	0	0.901	0.905	0.929	0.149	1164.1	20.9	130.33
3.6	3.0	4.1	2	0	1	0.891	0.908	0.921	0.181	280.6	29.7	130.84
3.5	3.0	3.4	2	0	2	0.898	0.894	0.917	0.203	167.8	41.3	134.46
3.9	1.5	3.8	2	1	0	0.902	0.909	0.929	0.152	1344.0	34.1	134.09
3.0	2.0	2.2	2	1	1	0.842	0.751	0.908	0.325	115.2	39.4	163.41
3.6	2.1	3.2	2	1	2	0.860	0.863	0.915	0.255	219.3	69.5	154.17
4.0	2.2	3.7	2	2	0	0.911	0.837	0.930	0.210	119.7	48.1	134.79
3.1	1.5	2.2	2	2	1	0.884	0.792	0.920	0.303	99.0	79.5	179.58
2.7	1.4	3.4	2	2	2	0.844	0.773	0.871	0.319	445.1	55.4	162.11

TABLE 5.9: OPTIMAL SOLUTION OBTAINED FROM SIMULATION
(EQUIPMENT IS IN CAN BE REPAIRED STATE
IF IT IS WAITING FOR NEXT MAJOR REPAIR)

****

N(1) = 2	TS(1) = 3.815
N(2) = 0	TS(2) = 2.962
N(3) = 0	TS(3) = 3.556
TOTAL COST = 130.33	

****

TABLE 5.10: OPTIMAL SOLUTION OBTAINED FROM SIMULATION
(NO "CAN BE REPAIRED STATE")

****

N(1) = 0	TS(1) = 4.123
N(2) = 0	TS(2) = 2.895
N(3) = 0	TS(3) = 3.095
TOTAL COST = 108.40	

****

[illegible] VARIABLES USED IN SIMULATION FLOWCHART.

- [illegible] : SIMULATION TIME SCALE.
- [illegible] : TIME FOR NEXT EVENT OF ITEM J.
- [illegible] : PRIORITY INDEX OF ITEM J.
- [illegible] : TYPE SCHEDULED EVENT OF ITEM J.
- [illegible] : STATE INDEX OF REPAIR FACILITY.
- [illegible] : STATE INDEX OF EQUIPMENT J.
- [illegible] : AVERAGE TOTAL COST TILL PRESENT EVENT.
- [illegible] : AVERAGE TOTAL COST TILL PREVIOUS EVENT.
- [illegible] : TOTAL COST TILL PRESENT EVENT.
- [illegible] : TOTAL TIME FOR WHICH THE REPAIR FACILITY HAS BEEN IN IDLE STATE SINCE THE BEGINNING OF
- [illegible] : TOTALTIME FOR WHICH EQUIPMENT J HAS WAITED IN QUEUE SINCE THE BEGINNING OF SIMULATION.
- [illegible] : PRIORITY INDEX FOR ITEM J.
- [illegible] : TIME WHEN THE EQUIPMENT CAME OUT AFTER REPAIR.
- [illegible] : DURATION OF MINOR REPAIR FOR EQUIPMENT J.
- [illegible] : DURATION OF MAJOR REPAIR FOR EQUIPMENT J.
- [illegible] : DURATION OF BREAKDOWN REPAIR FOR EQUIPMENT J.

equipment is getting repaired. This causes an additional downtime cost for the system. When the operation durations are small, this additional downtime cost contribution in the total cost of the system, is very significant.

2. Chance Failures:

Inclusion of chance failures, along with upsetting cost for the system. This effect of chance failures is very much dominant when total operation duration between two consecutive major repairs is very large, (that is $N_j\ d_j$ is very high).

3. Can be Repaired State:

From the definition of can be repaired state given in Sec. 5.2, it is obvious that i_j can have only integer variables. This is a serious limitation for our solution procedure because the optimal value of i_j, in most of the cases, may not be an integer. Deciding whether an equipment has 'can be repaired' state or not, should depend upon the effective age of the equipment and the time for which it has to remain in nonoperating state before it can get its turn for repair.

Besides the points mentioned above, there are many other factors which cause the simulation results to be very much different from that of approximate-joint-repair-scheduling problem. A joint repair policy does not depict

the picture of repair schedule very accurately because it does not account for the cases when any two or more, but not all, equipments have simultaneous major repairs.

Inspite of all the discrepancies between the two results, it is always economical to solve the parallel joint repair scheduling problem for obtaining initial hints about the optimal solution . The effect of chance failures and limited repair facilities is to increase the operation durations for all the equipments for given set of N_j's, we, therefore, can adjust our initial interval of uncertainity accordingly before running the actual simulation problem.

5.10 Scope for Future Studies:

In Sec. 5.3.1 we have mentioned four possible performance measures for our system. One can think of many more similar performance measures. For our present study we concentrated only on the total cost per unit time, but any combination of these performance measures can be used to provide a suitable single performance measure for future studies.

Many of the performance measures mentioned in Sec.5.3.1 can be considered as constraints on the system. We can impose a constraint that availability of particular equipment, or availability of the system as a whole should not fall below a particular predefined level. Our problem, under such circumstances, will be somewhat parallel to a constrained optimization problem.

Instead of dealing with a single repair facility system one can consider a system with multi-repair facilities with identical or different repairing capabilities.

CHAPTER VI

PARAMETRIC ANALYSIS

6.1 Introduction:

In this chapter we will study the effect of various parameters on the optimal values of the objective function and decision variables. Since all the decision variables (N_j's and m_j's) except the basic time period, are integers, the parametric analysis will be very much difficult even for very simple systems. Keeping in view the complications involved in it we will concentrate only on the systems where all the equipments can be treated independently (that is, the situation described in Chapter I). We will study the effect of following parameters:

1. Duration of each minor repair [d_j]
2. Duration of each major repair [d_M]
3. Cost of each minor repair [P_j]
4. Cost of each minor repair [AS_j]
5. Downtime cost rate [R_j]

6.2 Effect of Input Parameters on Decision Variables:

Figs. 6(a) - (c) shows the effect of various input parameters on the decision variables. Related input data is

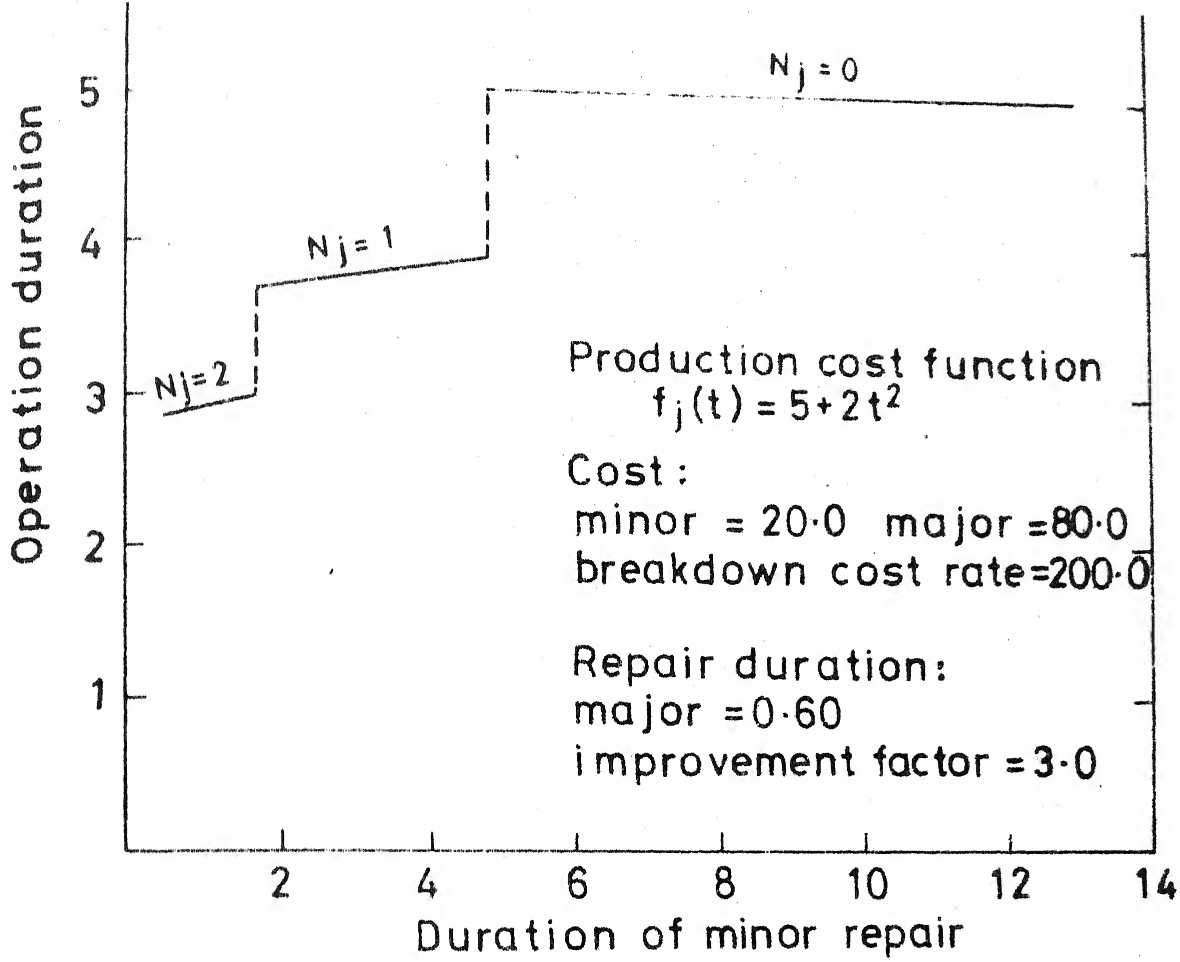
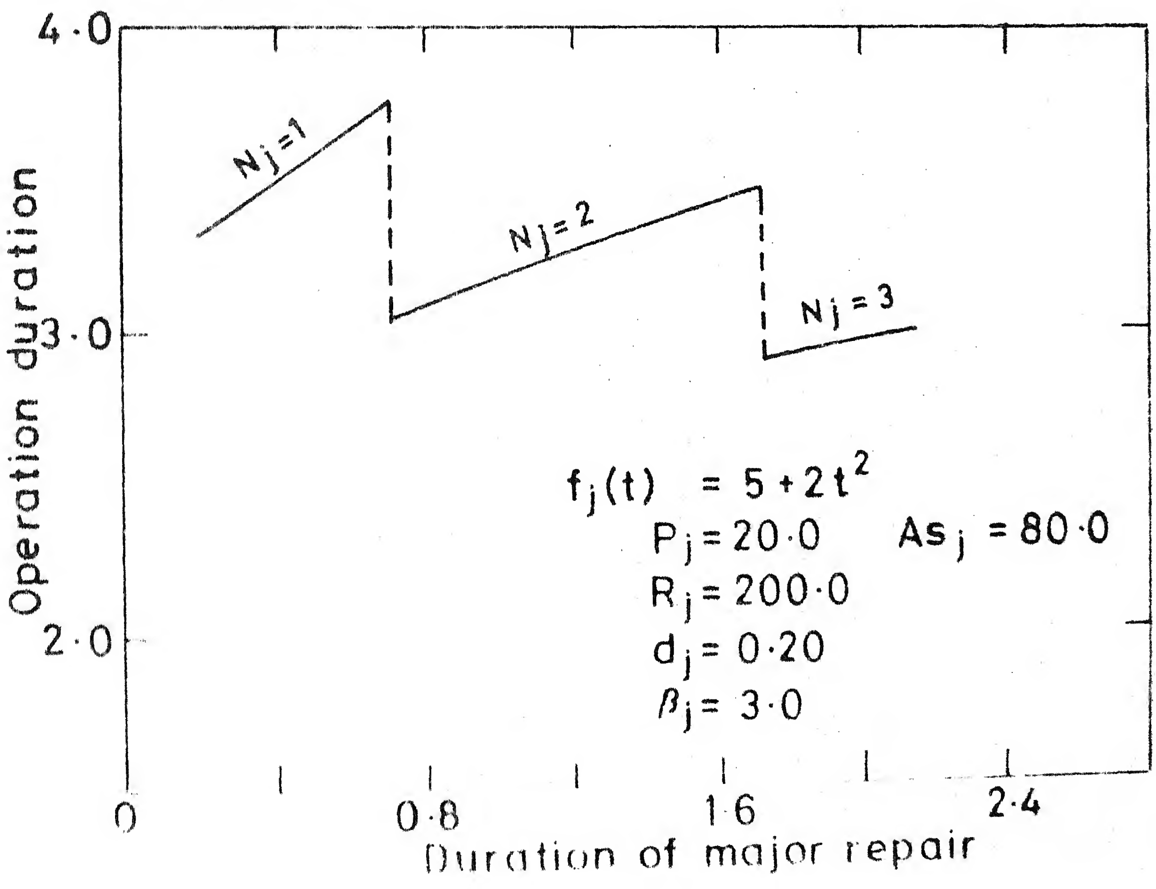


Fig. 6·1(a) Effect of minor repair duration



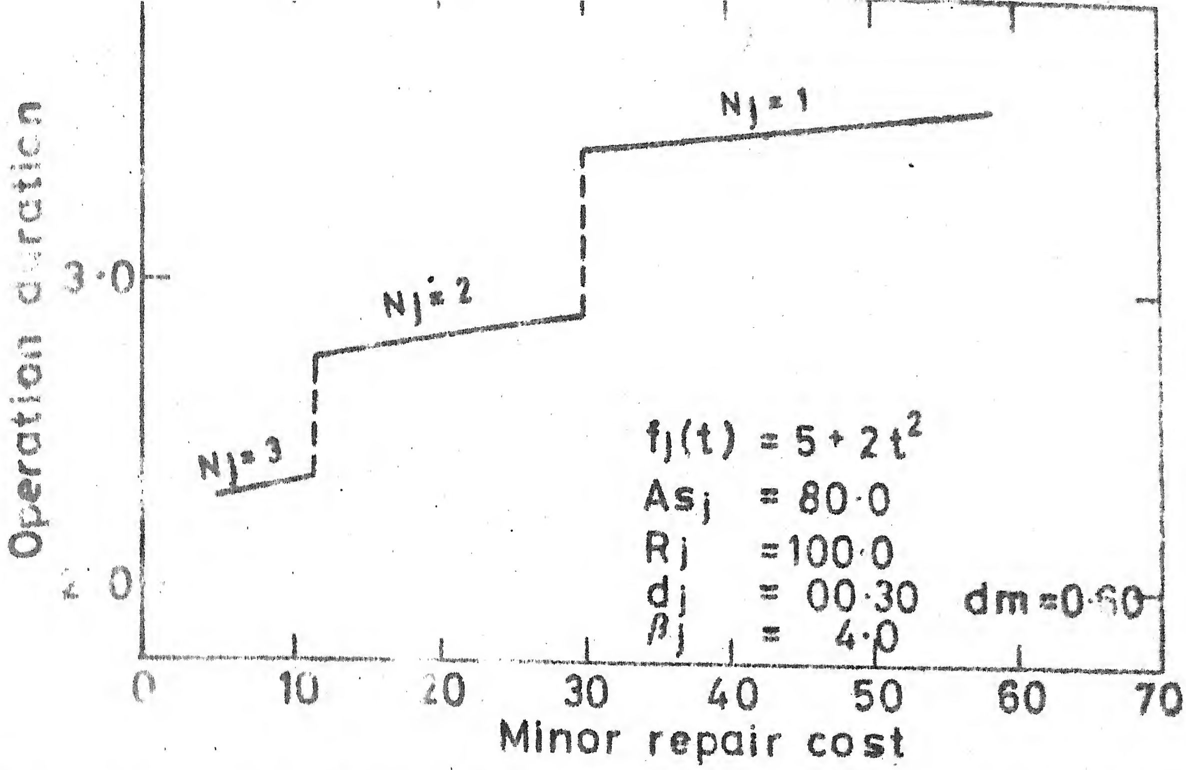


Fig. 6·1(c) Effect of minor repair cost

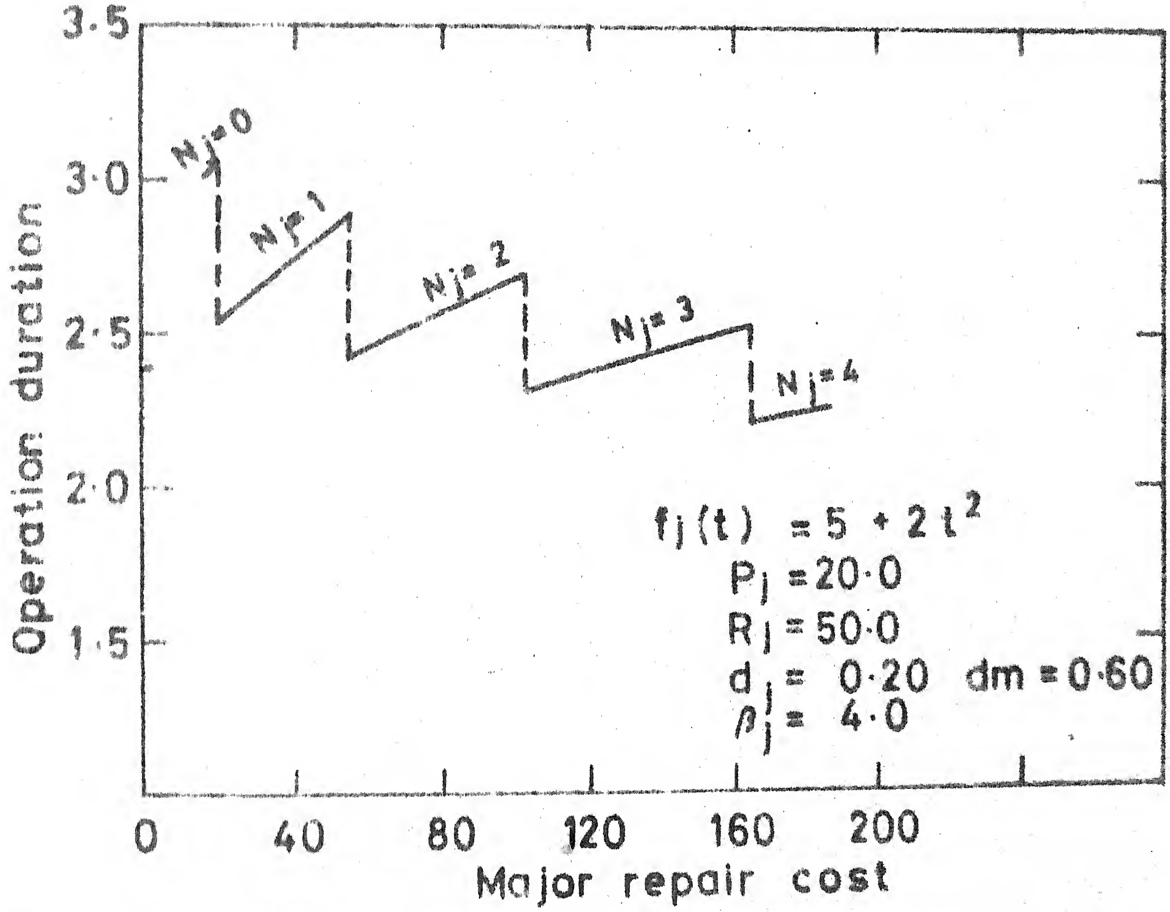


Fig 6 1 (d) Effect of major repair cost

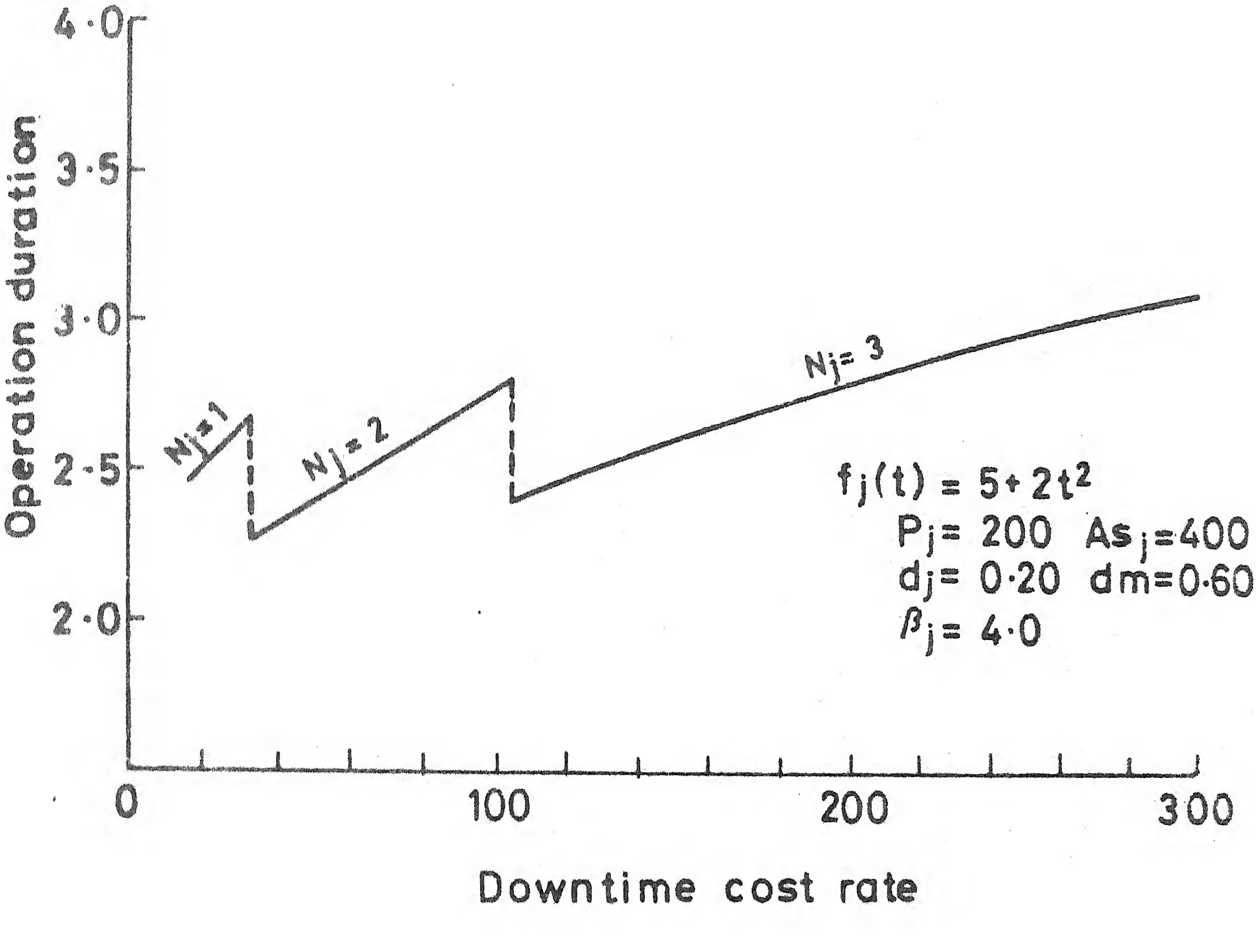


Fig 6·1(c) Effect of downtime cost rate

given on the corresponding figures. Following observations can be made from these figures:

(a) Effect of Duration of Minor Repair:

Any increase in the minor repair duration can affect the scheduling of repairs in two ways:

1. With any increase in the minor repair duration, if the optimal number of minor repairs between two consecutive major repairs does not change, the operation duration between any two consecutive repairs goes up. It can also be noted from Fig. 6.1 that, the rate of increase in operation duration between any two consecutive repairs, with increase in d_j is large for high value of N_j and small for small value of N_j that is,

$$\frac{\partial t_j}{\partial d_j}\Big|_{N_j} \geq \frac{\partial t_j}{\partial d_j}\Big|_{N_j'} \quad \text{for } N_j \geq N_j'$$

2. Any increase in the minor repair duration also tries to decrease the number of minor repairs between two consecutive major repairs. The decrease in number of minor repairs between two consecutive major repairs is usually accompanied with a sudden increase in the operation duration between any two consecutive repairs. This is also noted that, this increase in operation duration between any two consecutive repairs is small for large N_j and large for small N_j, where N_j is the number of minor repairs between any two consecutive repairs.

This is also noted that, this increase in operation duration between any two consecutive repairs is small for large N_j and large for small N_j, where N_j is the number of minor repairs between any two consecutive repairs.

[We should also keep in view that the curves shown in Fig. 6.1, though very close to straight lines, are not straight lines, that is, $\partial t_j/\partial d_j \,|_{N_j}$ are not a constant].

(b) Effect of Major Repair Duration:

Any change in major repair duration also affects the scheduling of repairs in two ways:

(1) Any increase in the major repair duration, causes the operation duration between two consecutive repairs to go up, provided the number of minor repairs between two consecutive major repairs does not change. This rate of increase in operation duration with respect to change in major repair duration, is large for small N_j and small for large N_j. This can also be written as:

$$\frac{\partial t_j}{\partial d_{M_j}}\Big|_{N_j} \geq \frac{\partial t_j}{\partial d_{M_j}}\Big|_{N_j'} \quad \text{for } N_j' \geq N_j$$

(2) Any increase in the major repair duration also tries to increase the number of minor repairs between two consecutive major repairs. This change is number of minor repairs between two consecutive major repairs is usually accompanied with a decrease in the operation duration between any two

two consecutive repairs is small for large N_j and large for small N_j.

(c) Effect of Minor Repair Cost:

Effect of any change in minor repair costs on the scheduling of repairs is similar to the effect of change in minor repair durations.

(d) Effect of Major Repair Cost:

Effect of any changes in major repair costs is similar to the effect of change in major repair duration.

(c) Effect of Downtime Cost Rate:

The effect of downtime cost rate on the optimal scheduling depends on the values of the minor and major repair costs and durations. However, following observations about the effect of downtime cost rate on the repair scheduling can be made, when all other parameters are kept at a constant level.

(1) As the value of downtime cost rate goes up the optimal operation duration between any two consecutive repairs goes up. When the ratio of major repair cost and the minor repair cost is very much large compared to the ratio of the major repair duration and minor repair duration then the rate of increase in optimal operation duration with downtime cost rate is large for large values of N_j and small for small value

of N_j. On the other hand when the ratio of the major repair cost and minor repair cost is very small compared to the ratio of major repair duration and minor repair duration the rate of increase in the optimal operation duration between any two consecutive repairs with downtime cost rate is small for large values of N_j and large for small values of N_j.

(2) Effect of downtime cost rate on the number of minor repairs between two consecutive major repairs is as follows.

(a) If the ratio of major repair cost and the minor repair cost is very large compared to the ratio of major repair duration and minor repair duration, then the effect of R_j on the repair scheduling is similar to the effect of minor repair cost.

(b) If the ratio of major repair cost and the minor repair cost is very small compared to the ratio of major repair duration and the minor repair duration, then the effect of downtime cost rate on the repair scheduling is similar to the effect of major repair cost.

(c) When the above mentioned two ratios are very close to each other then the effect of downtime cost rate on the number of minor repairs between two consecutive major repairs is negligible.

6.3 Explanations for the Effect of Parameters on the Repair Scheduling:

To explain the effect of various parameters, mentioned above, on the repair scheduling we introduce following new variables:

(1) Factor (1) $= R_j \, d_j + P_j$

(2) Factor (2) $= R_j \, d_M + AS_j$

(3) Ratio (1) $=$ Factor (1)/factor (2)

Here, factor (1) is the total cost encurred in one minor repair and factor two is the total cost encurred in one major repair. As a general rule any increase in the factor one or factor two (i.e. the total repair cost for minor repair or major repair) will always result in a larger operation duration between two consecutive repairs and, consequently, a smaller number of total repairs per unit time. Any increase in ratio one can be thought as a relative increase in the total cost for one minor repair as compared to the total cost of one major repair. Obviouslyan increase in ratio one will result in larger number of minor repairs between two consecutive major repairs and a decrease in ratio one will have the reverse effect.

Let us now try to explain the effect of each of the parameters discussed above.

(a) Effect of Minor Repair Duration:

When all the parameters are kept unchanged and only the minor repair duration is increased then this will cause an increase in factor one and ratio one. As a result of increase in factor one, the operation duration between any two consecutive repairs will go up. Now, for a small value of N_j, the downtime cost due to minor repairs is much smaller than the downtime cost component due to major repairs, for small values of N_j, therefore, a particular change in minor repair duration (say from d_j to $d_j + \Delta d_j$) will have smaller effect on the total cost per unit time than the effect produced by the same change in d_j for higher value of N_j. This explains why the optimal value of t_j changes rapidly with any change in d_j for large N_j and slowly for small N_j. For the extreme case, when the number of minor repairs between two consecutive major repairs is zero (i.e. $N_j = 0$), any change in minor repair duration does not have any effect on the total cost per unit time. The operation duration between two consecutive repair, under such situations does not change with change in minor repair duration.

Any increase in ratio (1) emplies that the ratio of total cost for one minor repair to the total cost for one major repair has gone up. As a result of this any increase in ratio one will have a tendency of decreasing the number of

minor repairs between two consecutive major repairs. This explains the decreasing tendency of N_j , between optimal value of t_j for given N_j.

Figure 6.2 depicts variations in total cost per unit time with changes in t_1 and N_1 for equipment 1, in example of Chapter II. We observe following relations:

(1) The value of the optimal t_j for given N_j keeps decreasing with any increase in N_j, that is,

$$t_j\ (N_j)\ \geq\ t_j\ (N_j)$$

$$\text{for } N_j\ \leq\ N_j$$

(2) It can also be noted that the sudden change in the value of t_j, due to change in number of minor repairs between two consecutive major repairs from N_j to $N_j + 1$, is larger for smaller N_j and smaller for larger N_j. This can be written as,

$$t_j^*(N_j) - t_j^*(N_j+1)\ \geq\ t_j^*(N_j') - t_j^*(N_j' + 1)$$

$$\text{for } N_j\ \leq\ N_j'$$

These two observations clearly explain why there is a sudden increase in the optimal operation duration due to an decrease in N_j. These also explain why this sudden increase in t_j, due to change in the optimal number of minor repairs between two consecutive major repairs, is higher for lower N_j and lower for higher N_j.

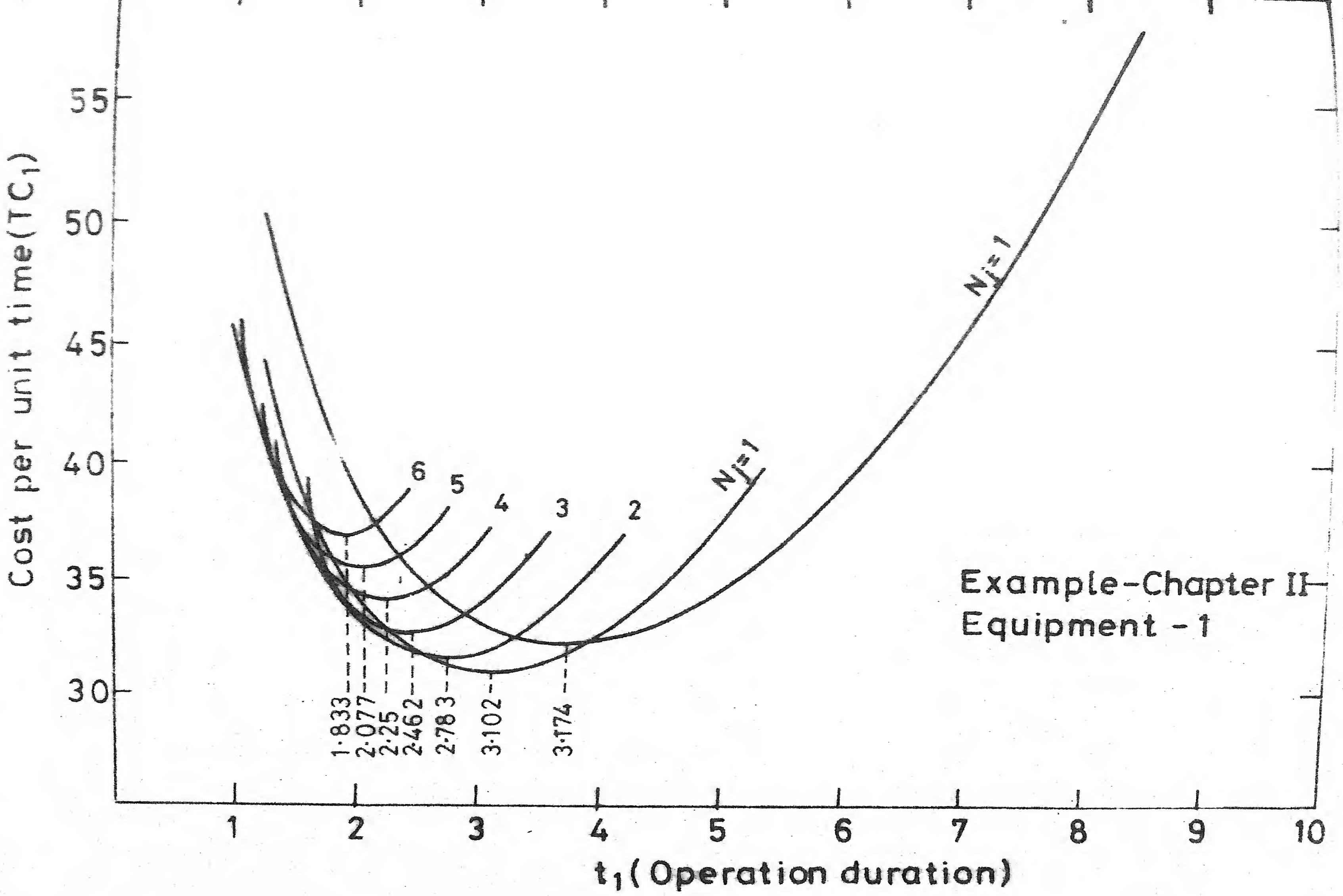


Fig. 6·2 Variation in total cost per unit time with changes in N_1 and t_1

(b) Effect of Major Repair Duration:

From the definition of the factor (1), factor (2) and ratio (1), we can see that any increase in the major repair duration will have no effect on factor one but it will cause an increase in factor two and decrease in ratio (1). Any decrease in ratio (1) will tend to increase the number of minor repairs between two consecutive major repairs. The effect of increase in factor (2) is to increase the operation duration between any two consecutive repairs, provided the value of N_j does not change. This explains the tendency of increase in N_j and decrease in t_j (for given N_j).

Since for small values of N_j, the major component of the downtime cost is due to the major repair only, therefore a particular change in major repair duration (say from d_M to $d_M + \Delta d_M$) affect the total cost per unit time, much more for smaller N_j than for larger N_j. This explains why the change t_j caused by change in d_M, for constant value of N_j, is very rapid for smaller N_j and very slow for larger N_j.

While explaining the effect of minor repair duration on the scheduling of repairs, we made two observations, based on the example in Chapter I, regarding the relation between the optimal operation duration and the optimal number of minor repairs between two consecutive major repairs. Now with the help of the first observation the sudden decrease in the optimal

operation duration between two consecutive repairs accompanied with any increase in N_j can be easily understood. The second observation helps us to explain why the sudden decrease in t_j is, high for higher number of minor repairs between two consecutive major repairs, and low for smaller number of minor repairs between two consecutive major repairs.

(c) Effect of Minor Repair Cost:

The effects on the values of factor (1) and ratio (1) due to any change in P_j is similar to the effects produced by the change in d_j. All the changes in the repair scheduling due to changes in P_j can, therefore, be explained on the lines parallel to the those, followed while explaining the effects of changes in d_j.

(d) Effect of Major Repair Duration:

Here again, it can be seen that the changes in AS_j affect the values of factor (2) and ratio (1) in the same way as the changes in d_M do. As a result of this, any effect of the changes in AS_j on the repair scheduling can be explained in the same way as it was done for the case of effects due to the changes in d_M.

(e) Effect of Downtime Cost Rate:

Any increase in the value of R_j always increases the values of factor (1) and factor (2) and thus causes the optimal

operation duration between any two consecutive repairs to go up. The effect of R_j on ratio one depends on the relative values of AS_j, P_j and d_M, d_j. If the ratio of AS_j and P_j is more than the ratio of major repair duration and the minor repair duration then any increase in R_j will try to increase the ratio one and consequently decrease the number of minor repairs between any two consecutive major repairs. Under such circumstances the effect of increasing and decreasing R_j on the repair scheduling is similar to the effect of increasing or decreasing the minor repair cost.

On the other hand when the ratio AS_j/P_j is less than the ratio d_M/d_j then any increase in R_j will try to decrease the ratio one and consequently increase the number of minor repairs between any two consecutive major repairs. Under these circumstances the effect of increasing or decreasing R_j, on the optimal operation duration and the optimal number of minor repairs between two consecutive major repairs, is same as the effect of increasing or decreasing the major repair cost.

When the ratio AS_j/P_j and d_M/d_j are nearly equal than any increase in R_j will only cause an increase in the optimal operation duration between any two consecutive repairs. The effect of changes in R_j on the optimal number of minor repairs between any two consecutive major repairs will be nearly

negligible. This is because N_j is a discrete variable and, therefore, it will not be affected by small change in ratio (1). Since, under the condition mentioned above, any change in the downtime cost rate will have very little effect on the value of ratio (1), the value of N_j in most of the cases, will not get affected by this change in R_j. This completely explains the effect of changes in R_j on the optimal repair scheduling.

CHAPTER VII

CONCLUSIONS

The present study was proposed for minimization of cost, by optimal scheduling of repairs, for preventive maintenance of equipments in either a goods producing system or a service producing system. Models and solution procedures were developed for both kinds of systems under the assumptions that the deteriorations of the equipments with respect to time and improvements in them due to repairs are deterministic. Instead of considering total cost along with the availability of individual equipments or the system as a whole we concentrated on total cost alone. Solution procedures for obtaining optimal repair schedules were presented and their merits and demerits were discussed in the respective chapters. Some improvements in terms of incorporating nonavailability constraints for both kinds of production systems have also been suggested at the end of respective chapters. For service producing systems, as a consequence of complications in its formulation, only simplified solution procedures could be presented, although some improvements in the solutions, obtained by using these procedures, have been suggested for future studies at the end of IV Chapter. The case of

stochastic repair durations for simple goods producing systems, prone to both, wear-out and chance failures, have been considered in Chapter V. Because of the complications involved in mathematical formulation , we resorted to simulation techniques. Suggestions and improvements in these simulation techniques are also included in Chapter V.

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APPENDIX A

It has been claimed in Sec. 4.3.3.2 that for the optimum solution of a mixed repair scheduling problem it is a necessary condition that atleast for one of the equipments max $(0, m_jT - d_M - t_j (N_j + 1, 0))$ be zero. This can be proved by contradiction.

Let us assume that our optimal solution has basic time period equal to T and the total cost per unit time equal to K_1. Let us also assume that for all the equipments max $(0, m_jT - d_M - t_j (N_j + 1, 0))$ is non-zero. Let us name this solution as solution-one. We now construct an another feasible solution by decreasing T, slowly from its present value, till for the first time, at least for one of the equipment, max $(0, m_jT - d_M - t_j(N_j :1, 0))$ becomes zero. Let this decrease in T be denoted by ΔT. We name this new solution as solution-two. Even for solution two number of minor repairs between two consecutive major repairs for all the equipments are still the same.

It is obvious that if we are at all going for any repair policy then the cost incurred in following that repair policy will always be less than the cost when all the equipments are allowed to remain in down state throughout. Cost per unit time for the latter case is $\sum_{j=1}^{M} R_j$. This statement can be written as:

$$K_1 < T \sum_{j=1}^{M} R_j \quad .$$

If we can some how prove that our solution two is better than solution one then solution one can not be a global optimum since solution two is both feasible and better than this.

Let us start from the above inequality,

$$K_1 < T \sum_{j=1}^{M} R_j$$

$$\Delta T\ K_1 < \Delta T \cdot \sum_{j=1}^{M} R_j\ T$$

(this is permissible since ΔT is nonzero)

$$-\Delta T . K_1 > -\Delta T \sum_{j=1}^{M} R_j . T$$

$$K_1\ T - \Delta T\ K_1 > K_1\ T - \Delta T \sum_{j-1}^{M} R_j . T$$

$$K_1\ (T - \Delta T) > [K_1 - \Delta T \sum_{j=1}^{M} R_j\ ] . T$$

$$\frac{K_1}{T} > \frac{K_1 - \Delta T \sum_{j=1}^{M} R_j}{T - \Delta T}$$

Since K_1/T is total cost per unit time for the solution one and $[K_1 - \Delta T \sum_{j=1}^{M} R_j]/[T - \Delta T]$ is total cost per unit time for solution two (since $\Delta T \sum_{j=1}^{M} R_j$ is total cost reduction in downtime cost for the system and $T - \Delta T$ is the time period for our new solution). Thus the total cost per unit time for solution two is less than the total cost per unit time for solution one.

This completes the proof.